Development of a Halbach Array Magnetic Levitation System

Senior Project Report

Students:
Dirk DeDecker
Jesse VanIseghem

Advisors:
Mr. Steven Gutschlag
Dr. Winfred Anakwa

Date:
May 10, 2012
Abstract:

The need for an efficient magnetic levitation train system is rapidly growing. With the population of the Earth rising every day, roads are becoming more and more congested. Magnetic levitation train technology is being researched as a possible method to alleviate transportation problems. For this project, an improved Halbach array magnetic levitation system was developed to achieve a desired levitation height at a low rotational speed. This project made use of information found about a small scale model for a magnetic levitation train system. Theoretical analysis was performed using a MATLAB GUI and experimental work was performed in lab. A 5x25 Halbach array with 6 mm cube magnets was used to build the magnetic levitation device. A prototype rotary track made from a sheet of copper with conducting strips was developed for the system. The track was mounted on a polyethylene wheel and was driven by a motor to simulate the effect of a train travelling across an actual track. The results obtained in this project will permit the ECE department to build a larger scale system in the future.
# Table of Contents

- Project Summary ................................................................. 1
- Changes to Original Proposal ............................................. 1
- Previous Work Completed ................................................... 2
- Physics of the Inductrack System ........................................ 3
- Design ................................................................................. 6
- Simulation ............................................................................. 7
- Fabrication ........................................................................... 8
- Experimental Setup ............................................................. 12
- Displacement Sensor Calibration ........................................ 14
- Experimental Results ........................................................... 16
- Discussion of Results ........................................................... 20
- Equipment ............................................................................ 23
- Acknowledgments ............................................................... 24
- Patents .................................................................................. 25
- References ............................................................................. 26
List of Tables and Figures

Figure 1: Halbach Array.................................................................3
Figure 2: R-L Circuit Representation of Inductrack.........................3
Figure 3: Force Diagram..............................................................6
Figure 4: Matlab GUI for Simulation...........................................8
Figure 5: Fabricated Copper Track..............................................9
Figure 6: Dremel Setup for Carving out the Balsa Wood...............10
Figure 7: Balsa Wood Carved out to House Magnets.....................10
Figure 8: Halbach Array Device with Magnets Glued....................11
Figure 9: Halbach Array Device with Protective Shield...............11
Figure 10: Motor Stand and System Set Up.................................12
Figure 11: Experimental Set up with Shield and Displacement Sensor..................................................13
Figure 12: Displacement Sensor Output Data................................14
Figure 13: Plot of Levitation Force vs. Velocity..........................17
Figure 14: Plot of Levitation Height vs. Velocity.........................19
Table 1: Displacement Sensor Output..........................................15
Table 2: Trial 1 Force Sensor Data................................................17
Table 3: Trial 2 Force Sensor Data.......................................................17
Table 4: Trial 3 Force Sensor Data.......................................................18
Table 5: Displacement Data.................................................................19
Table 6: Drag Force Calculations.........................................................22
Table 7: Purchased Equipment.............................................................24
**Project Summary:**

The goal of this project is to demonstrate successful levitation of a Halbach array magnetic device using the Inductrack method. The project is an extension of projects completed in previous years, building on Paul Friend’s 2004 project [1] and Glenn Zomchek’s 2007 project [2]. These projects were able to show successful levitation, but only to a maximum height of 0.45 mm. These projects and their findings were used to begin this project and their results were used for comparison.

This year, all of the parameters were recalculated. This led to a new experimental set up, including the magnets, Halbach array, Maglev device, wheel, and motor. A 5 by 25 Halbach array was used for testing. The project began with a goal of achieving 0.5 cm of levitation at a track speed of 10 m/s.

**Changes to Original Proposal:**

In the original proposal, the project included design and implementation of a closed loop controller for levitation height. Half way through the year, the decision was made to change the closed loop system to an open loop system. The design, machining, and construction of the entire system took longer than expected. In order to design the controller, the entire system would have to be modeled. Testing of the system did not take place until April, leaving no time for modeling and controller design. Furthermore, due to the necessity of having two people working on the fabrication, it was difficult to divide the work and give one student the task of building the system and the other student the task of theoretical modeling. In the end, changing to the open loop system gave ample time to ensure that the design would meet the specifications.
**Previous work completed:**

Dr. Sam Gurol and Dr. Post have worked on “The General Atomics Low Speed Urban Maglev Technology Development Program” utilizing the rotary track and Inductrack methods [5]. This program is working to design an urban transportation system for the United States that will use the Inductrack to levitate the trains at low speed.

Dr. Richard Post was the head scientist for the magnetic levitation program at Lawrence Livermore National Laboratory. Dr. Post pioneered the Inductrack method of magnetic levitation in the 1990’s [3]. The Inductrack method is now being researched by NASA for launching rockets into space.

Previous work has also been completed by Bradley University electrical engineering students in previous senior projects. The first of these projects was by Paul Friend in 2004. Paul Friend helped to obtain all the levitation equations and wrote the code for the Matlab GUI to run simulations. In 2007, Glenn Zomchek designed a Maglev system using the rotary inductrack method. Zomcheck obtained successful levitation to 0.45 mm.

The previous work of individuals on magnetic levitation was of great assistance while working on the magnetic levitation project this year.
Physics of the Inductrack System:

For the project, the Halbach array was be used to create a strong magnetic field for levitation. The Halbach array is shown in figure 1.

The Halbach array was designed by Klaus Halbach. The arrangement of the polarities of the magnets creates a strong magnetic field on one side of the array, while canceling most of the magnetic field on the other side. The peak strength of the array is given by:

$$B_0 = B_r (1-e^{-kd}) \sin(\pi/M) / (\pi/M) \text{ Tesla}$$

(1)

where $k = 2\pi/\lambda$, $M$ = # of magnets, $B_r$ = magnet strength, $d$ = thickness of each magnet [3].

$\lambda$ = wavelength of the Halbach array

The track will be of the Inductrack design, using close-packed conductors, made utilizing thin aluminum or copper sheets. This design allows for levitation at low velocities. The Inductrack can be modeled as an R-L circuit, shown in figure 2. The transfer function will have a pole at $-R/L$.

Figure 1: Halbach Array

Figure 2: R-L Circuit Representation of Inductrack
The Halbach array moving at velocity, \( v \) [m/sec], over the inductrack generates flux linking the circuit. The flux is \( \phi_0 \sin(\omega t) \), where \( \phi_0 \) [Tesla-m^2] and:

\[
\omega = \frac{2\pi}{\lambda} v \text{ rad/sec} \quad (2)
\]

The voltage induced in the inductrack circuit is the rate of change of flux given by:

\[
V(t) = \omega \phi_0 \cos(\omega t) \quad (3)
\]

From figure 3, the Inductrack R-L circuit current equation is given by:

\[
V(t) = L \frac{di(t)}{dt} + Ri(t) \quad (4)
\]

Dr. Post and Dr. Ryutov [3] used the induced current, \( i(t) \), which is the solution of (4), and magnetic field to derive equations for the lift force and drag force.

Lift force:

\[
<F_y> = B_0^2 w/2 k_L \left[ 1/(1+(R/\omega L)^2) \right] e^{-ky_1} \quad (5)
\]

Drag force:

\[
<F_x> = B_0^2 w/2 k_L \left[ (R/\omega L) / [1+(R/\omega L)^2] \right] e^{-ky_1} \quad (6)
\]

where \( y_1 \) is the levitation height in meters.

The phase shift of the system relates to drag and levitation forces.

\[
\text{Lift/Drag} = \omega L/R \quad (7)
\]

To maximize lift, a large amount of inductance and low resistance is desired. The inductance of the track is given by the equation:

\[
L = \frac{\mu_0 w}{2kd_c} \quad (8)
\]

where \( d_c \) is the center to center spacing of conducting strips and \( w \) is the track width.

The equation for \( L \) shows that it is desired to have the narrow transverse slots on the track as wide and close together as possible to maximize \( L \).

The equation for resistance is given by:

\[
R = \frac{PcRc}{(Nt*c*Ns)} \quad (9)
\]

Where \( Rc \) is the resistivity.
The equation for $R$ shows that it is desired to have a low resistivity in order to lower the resistance and increase the lift/drag ratio.

The force needed to levitate the device is given by:

$$F = m \times 9.81 \text{ Newtons} \quad (10)$$

where $m$ is the mass of the device.

Solving Lift/Drag equation (7) for velocity, the breakpoint velocity of the system is given by:

$$v_b = \frac{\lambda \omega}{2\pi} \text{ m/sec} \quad (11)$$
Design:

The first parts of the system that needed to be designed were the wheel and track. It was decided to use a 9”, or 22.86 cm, radius wheel for the project. Using a wheel of this size allowed for high tangential speeds at low motor RPM. This size of wheel was also desired in order to allow for very high tangential speeds to be reached. It was decided to use a 1” motor shaft with a 3/16” keyway in order to strengthen the connection of the motor to the wheel.

The track was designed to allow for levitation at low speeds. As shown in equation (8), it is desired to have narrow slots on the track as close together as possible to maximize the lift/drag ratio. It was decided to have 1 mm slots on the track, with 4 mm spacing center to center. The slots were narrow enough to increase L, but not too narrow to be unable to fabricate. Resistance of the track is desired to be as low as possible. In past years, the inductrack has been manufactured out of aluminum. It was decided that copper would be used to make the track in order to decrease resistance. Copper has a resistivity of $1.68 \times 10^{-8} \Omega \cdot m$, whereas aluminum has a resistivity of $2.82 \times 10^{-8} \Omega \cdot m$. Comparing the results of simulations with these different parameters, it was found that the lift/drag ratio using aluminum was 0.102, and the lift/drag ratio using copper track was 0.171. This 67.6% increase in the lift/drag ratio made it worth buying the more expensive copper instead of aluminum.

Using equation (8), the inductance of the track can be found.

$$\mu_0 = 4\pi \times 10^{-7}$$
$$dc = 0.004 \text{ m}$$
$$\lambda = 0.028 \text{ m}$$
$$Pc = 0.1076 \text{ m}$$

$$L = 4\pi \times 10^{-7} \times 0.1076 / (4\pi \times 0.004 / 0.28)$$

$$L = 7.532 \times 10^{-8} \text{ H}$$

Using equation (9), resistance of the track can be found.

$$Rc = 1.68 \times 10^{-8} \Omega \cdot m$$
$$Nt = 0.003 \text{ m}$$
$$Ns = 1.0$$
$$c = 0.003175 \text{ m}$$

$$R = 0.1076 \times 1.68 \times 10^{-8} / (0.003 \times 0.003175 \times 1.0)$$

$$R = 1.9 \times 10^{-5} \Omega$$
The next part that had to be designed was the Halbach array device. It was decided to make the array using 6mm cubed magnets. These magnets were strong, but with a small mass, allowing for higher levitation at lower speeds. The array had a wavelength of 28mm. The force diagram for the setup is shown in figure 4.

\[ F_v = F_i \cos(\Theta) \]  \hspace{1cm} (12)

It was decided that approximately 90% of the forces created by the maglev system should be in the vertical direction. By using an angle of 25 degrees,

\[ \cos(25^\circ) = 0.9063 \]

Therefore, the angle of 25 degrees will keep 90.63% of the magnetic levitation force in the vertical direction.

In order to determine the number of magnets that will be used to make the maglev device, the arc length corresponding to an angle of 25 degrees on an 18” wheel.

The equation for arc length is given by

\[ s = r^* (\Theta) \]  \hspace{1cm} (13)

where \( \Theta \) is in radians.

Figure 3: Force Diagram

From figure 4,
Solving equation (13), it was found that the arc length is 3.93 inches to either side of the vertical. This allowed for a total array size of approximately 7.86 inches. It was determined that this arc length would allow for an array length of 25 magnets. The array, including glue, had an arc length of approximately 8”, which is what was specified by the computations.

In order to make the array the approximate width of the 2” track, the array was designed to be 5 magnets wide. In total, the array was designed to be 5x25 magnets.

Using equation (1), the strength of the Halbach array can be found.

\[
\begin{align*}
\text{Br} &= \text{strength of one magnet} = 1.21 \text{ tesla} \\
M &= \text{number of magnets per wavelength} = 4 \\
d &= \text{thickness of each magnet} = 0.006 \text{ m} \\
\lambda &= \text{wavelength} = 0.028 \text{ m}
\end{align*}
\]

Plugging these numbers into the equation,

\[
B_0 = 1.21 \times (1 - e^{-2\pi \times 0.006/0.028}) \times \sin(\pi/4)/(\pi/4)
\]

\[
B_0 = 0.8060 \text{ Tesla}
\]

Using equation (5), equation (2), and the values calculated for \( L \), \( R \), and \( B_0 \), the theoretical levitation force can now be calculated. The values will be calculated with \( y = 0.0085 \text{ m} \), which was where the device sat during the force measurement trials.

At 10 m/sec, \( \omega = (2 \times \pi/0.028) \times 10 = 2244 \text{ rad/sec} \)
\( A = 0.034 \times 0.174 = 0.005916 \text{ m}^2 \)
\( w = 0.034 \text{ m} \)

\[
f_y(\omega = 2244 \text{ rad/sec, } y = 0.0085) \\
= [0.8060^2 \times 0.034/(4 \times \pi \times 7.572 \times 10^{-8} \times 0.004/0.28)] \times [1/(2244 \times 7.53 \times 10^{-8}) \times (1 + (1.9 \times 10^{-5} / (2244 \times 7.53 \times 10^{-8}))^2)] \times 0.005916 \times e^{-4 \pi \times 0.0085/0.028}
\]

\[
f_y = 21.0 \text{ N}
\]

Using equation (6), the theoretical value for drag force can be found.

\[
f_x(\omega = 2244 \text{ rad/sec, } y = 0.0085) = [0.8060^2 \times 0.034/(4 \times \pi \times 7.572 \times 10^{-8} \times 0.004/0.28)] \times [1.9 \times 10^5/(2244 \times 7.53 \times 10^{-8}) \times (1 + (1.9 \times 10^{-5} / (2244 \times 7.53 \times 10^{-8}))^2)] \times 0.005916 \times e^{-4 \pi \times 0.0085/0.028}
\]

\[
f_x = 2.4 \text{ N}
\]
Simulation:

Using the Matlab GUI, designed by Paul Friend, simulations were run while changing different parameters. Simulation with the final parameters is shown in figure 5.

![Matlab GUI for Simulation](image)

**Figure 4: Matlab GUI for Simulation**
Fabrication:

The wheel and track were machined by Tri-City Machining in Peoria, Illinois. In order for machining to be completed, a mechanical engineer, Harrison Cohen, assisted in formulating drawings in ProE and AutoCad. These drawings were used by Tri-City to write code to machine the parts. The track was machined using a water jet cutting tool. The wheel was cut on the water jet and then turned on a lathe. A picture of the track is shown in figure 6.

![Fabricated Copper Track](image)

After receiving the track and wheel from Tri-City Machining, the track was rolled using a sheet roller and placed around the wheel. Once the track was placed around the wheel, it was realized that the track was about ¼” short. When computing the length of the track, the circumference of the wheel was computed and used to determine the length of the track. The 1/8” thickness of the track was not taken into account, causing the track to be too short.

There were two solutions to this problem, either to cut a small piece of copper to place into the gap, or machine a small amount off the diameter of the wheel. With the help of Darren DeDecker and Caterpillar Inc., a small amount was taken off the diameter of the wheel using a CNC machine. After this small amount was removed, the track fit perfectly around the wheel.

Once the track fit around the wheel, holes were drilled approximately every 2” on the track in order to connect the track to the wheel. It was decided that ¾” long #4 flat head stainless steel screws would be used. Stainless steel was used so that it would not react with the magnetic field. The screws were flat head because they were intended to be countersunk into the track, allowing the magnetic levitation device to be placed very close to the track. However, once the countersunk holes were drilled, the track became very weak. There was an insufficient amount of copper on the sides of the holes to allow the screws to safely and securely attach the track without kinking and possibly cracking the copper sheet.

To fix the problem of the screws not being able to grab onto enough copper to safely hold the track, it was decided to use pan head screws with washers. The washers provided more surface area for the screws to tighten down the track. The track was placed around the wheel, holes were pre-drilled into the plastic, and the screws were inserted to attach the track.
To construct the Halbach array device, a piece of 2”x2”x12” balsa wood was cut to the correct length. Then, the correct curvature was drawn onto the wood and cut out. In order to carve out the space for the magnets, a Dremel was used in conjunction with a metal bar that would only allow for 7mm to be carved out of the wood. The Dremel setup is shown in figure 7. The balsa wood after the carving out is shown in figure 8.

Figure 6: Dremel Setup for Carving out the Balsa Wood

Figure 7: Balsa Wood Carved out to House Magnets

Once the balsa wood was carved out, the magnets had to be glued into the wood. Industrial strength epoxy was used to glue the magnets. In order to make it so the magnets did not have to be glued one at a time, shrink wrap was used to form each row of 5 magnets. The magnets still had to be glued together inside of the shrink wrap, otherwise they would flip around. Once all 25 rows of 5 magnets were made, the rows began to be glued into the wood. Each row had to be clamped down and left for at least 12 hours. If they were not left for at least this amount of time, the epoxy would not completely set and the magnets would come unglued. Only two rows of magnets could be glued at a time to leave room for the clamps. Plastic clamps with rubber feet were purchased so that the magnets did not stick to the feet of the clamps when the clamps were removed. This
turned out to be a very difficult process, since all the magnets had to be glued in in such as way that their polarity attempted to turn them around. Since only two rows of magnets could be glued at a time, the process of gluing the magnets ended up taking about three weeks. The balsa wood, with magnets glued in, is shown in figure 9.

![Figure 8: Halbach Array Device with Magnets Glued](image)

Since the magnets could easily come unglued from the wood and fly out during testing, it was decided that a protective shield should be placed over top to keep the magnets contained. A piece of aluminum duct was used for this shield. It was chosen because it was very thin, so it would not force the device to be placed higher off the track. Also, the aluminum would not interfere with the magnetic field. The balsa wood structure with protective shield is shown in figure 10.

![Figure 9: Halbach Array Device with Protective Shield](image)

The Halbach array device was mounted into the plastic housing used in previous years. This plastic housing contained bearings that allow the device to slide up and down on rods that were pressed into an aluminum plate. The total mass of the device and plastic housing was 465 grams. Using equation (10), the force needed to levitate the device was 4.56 newtons.

In order to assemble the system together, a stand had to be built to hold the motor, wheel, motor shaft, Halbach array device, and sensors. For the base of the stand, 4” wide aluminum beams were placed 2” apart. On top of the base was the support structure to raise the motor up to the desired position. The motor had to be high enough to place the shaft at least 9” above the base of the stand, allowing clearance for the wheel to turn. Mounts were also designed and cut to hold bearings for the shaft. An aluminum structure was cut and connected to these mounts to provide the height for the Halbach array device mount. The aluminum plate with the rods that held the Halbach array device was bolted to an aluminum beam that held the device above the wheel. The aluminum plate had a
hole drilled through it, allowing for the force and displacement sensor to be connected to perform measurements. Most of the system was bolted together, so holes had to be drilled using a drill press and then bolts put in to connect everything together. The stands were made out of steel beams, which Mr. Gutschlag welded together. The aluminum beam to hold the Halbach array device was clamped to the aluminum uprights. This allowed for ease of adjustment of the device’s positioning. The set up is shown in figure 11.
Experimental Setup:

Figure 10: Motor Stand and System Set Up
For the demo, a shield was built in front of the device in case any part should come apart. The set up with the shield and displacement sensor is shown in figure 11.

Figure 11: Experimental Set up with Shield and Displacement Sensor
Displacement Sensor Calibration:

In order to measure levitation height, a displacement transducer, model MLT002N3000B5C, was used. The transducer outputs a linear voltage change corresponding to a change in displacement. In order to find out the linear change of the transducer, it was necessary to take measurements of the voltage and displacement changes and determine a best fit line. The sensor was powered by 5V, and the output was measured using a multi-meter. The displacement was measured using a vernier caliper.

<table>
<thead>
<tr>
<th>Input Voltage [V]</th>
<th>Displacement [in]</th>
<th>Output Voltage [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.323</td>
<td>0</td>
<td>0.23</td>
</tr>
<tr>
<td>5.323</td>
<td>0.4</td>
<td>0.963</td>
</tr>
<tr>
<td>5.323</td>
<td>0.9</td>
<td>2.302</td>
</tr>
<tr>
<td>5.323</td>
<td>1.2</td>
<td>3.0432</td>
</tr>
<tr>
<td>5.323</td>
<td>1.5</td>
<td>3.877</td>
</tr>
<tr>
<td>5.323</td>
<td>1.7</td>
<td>4.398</td>
</tr>
<tr>
<td>5.323</td>
<td>2.1</td>
<td>5.327</td>
</tr>
</tbody>
</table>

The data in table 1 was inputted into a best fit line calculator. The plot of this data is shown in figure 10.

Figure 12: Displacement Sensor Output Data
Using the data measured with the displacement sensor, figure 10 shows that:

\[ V_{\text{out}} = 2.49213 \times \text{displacement} + 0.100227 \]  \hspace{1cm} (14)

Equation (14) was used to calculate levitation height when the system was tested.
Experimental Results:
Once the entire system was assembled, initial testing was completed using the force sensor. The results of the three trials are shown in tables 2-4.

### Table 2: Trial 1 Force Sensor Data

<table>
<thead>
<tr>
<th>Motor Voltage [V]</th>
<th>RPM</th>
<th>Force [N]</th>
<th>Velocity [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>136</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>20</td>
<td>184</td>
<td>2.5</td>
<td>2.2</td>
</tr>
<tr>
<td>25</td>
<td>253</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>30</td>
<td>316</td>
<td>5.6</td>
<td>3.8</td>
</tr>
<tr>
<td>35</td>
<td>382</td>
<td>7.7</td>
<td>4.6</td>
</tr>
<tr>
<td>40</td>
<td>453</td>
<td>9.3</td>
<td>5.4</td>
</tr>
<tr>
<td>45</td>
<td>524</td>
<td>11.1</td>
<td>6.3</td>
</tr>
<tr>
<td>50</td>
<td>598</td>
<td>12.2</td>
<td>7.2</td>
</tr>
<tr>
<td>52</td>
<td>627</td>
<td>12.9</td>
<td>7.5</td>
</tr>
<tr>
<td>53.5</td>
<td>649</td>
<td>13.3</td>
<td>7.8</td>
</tr>
</tbody>
</table>

Once the motor speed got to 650 RPM, the system began to vibrate badly. If the system were to vibrate enough, the Halbach array device could come into contact with the track and wheel, and the whole system could come apart. For safety reasons, the motor speed was not increased further until a shield can be built for around the system.

Much of the vibration was due to a wobble in the wheel because the wheel was not tight on the shaft. A thin strip of aluminum tape was placed on the motor shaft to eliminate some space and cause the wheel to wobble less. Trial 2 measurements were taken after this alteration.

### Table 3: Trial 2 Force Sensor Data

<table>
<thead>
<tr>
<th>Motor Voltage [V]</th>
<th>RPM</th>
<th>Force [N]</th>
<th>Velocity [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>136</td>
<td>2.2</td>
<td>1.6</td>
</tr>
<tr>
<td>25</td>
<td>258</td>
<td>3.6</td>
<td>3.1</td>
</tr>
<tr>
<td>35</td>
<td>388</td>
<td>7.5</td>
<td>4.6</td>
</tr>
<tr>
<td>45</td>
<td>529</td>
<td>11.3</td>
<td>6.3</td>
</tr>
<tr>
<td>52</td>
<td>628</td>
<td>13.5</td>
<td>7.5</td>
</tr>
<tr>
<td>55</td>
<td>679</td>
<td>14.2</td>
<td>8.1</td>
</tr>
<tr>
<td>59</td>
<td>741</td>
<td>14.9</td>
<td>8.9</td>
</tr>
</tbody>
</table>
The system showed less vibration during trial 2, however, as a safety precaution the motor was not increased past 741 RPM due to continuing vibrations.

For trial 3, the decision was made to measure motor current, allowing calculation of drag forces. These results are shown in Table 4.

**Table 4: Trial 3 Force Sensor Data**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>140</td>
<td>2.8</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>25</td>
<td>260</td>
<td>4.3</td>
<td>3.6</td>
<td>3.1</td>
</tr>
<tr>
<td>35</td>
<td>390</td>
<td>5.4</td>
<td>7.6</td>
<td>4.7</td>
</tr>
<tr>
<td>45</td>
<td>533</td>
<td>6</td>
<td>11.0</td>
<td>6.4</td>
</tr>
<tr>
<td>50</td>
<td>609</td>
<td>6.2</td>
<td>12.3</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Using the data in table 4, a plot of levitation force vs. velocity was made, which is shown in figure 11.

![Figure 13: Plot of Levitation Force vs. Velocity](image)
After data was recorded with the force sensor, the system was set up with the displacement sensor. By taping washers onto the outside of the wheel, the wheel was statically balanced to attempt to ease more of the vibrations. A thick piece of rubber was also placed under the motor stand to help absorb vibrations. The results from testing with the displacement sensor are shown in Table 6.

Table 5: Displacement Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.32</td>
<td>2.10</td>
<td>5.32</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>140</td>
<td>5.31</td>
<td>2.09</td>
<td>5.31</td>
<td>0.01</td>
<td>1.68</td>
</tr>
<tr>
<td>25</td>
<td>3.7</td>
<td>277</td>
<td>5.222</td>
<td>2.06</td>
<td>5.22</td>
<td>0.10</td>
<td>3.32</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>347</td>
<td>5.213</td>
<td>2.05</td>
<td>5.21</td>
<td>0.11</td>
<td>4.15</td>
</tr>
<tr>
<td>35</td>
<td>4.4</td>
<td>417</td>
<td>5.203</td>
<td>2.05</td>
<td>5.20</td>
<td>0.12</td>
<td>4.99</td>
</tr>
<tr>
<td>40</td>
<td>3.9</td>
<td>505</td>
<td>5.138</td>
<td>2.02</td>
<td>5.14</td>
<td>0.18</td>
<td>6.04</td>
</tr>
<tr>
<td>45</td>
<td>3.6</td>
<td>591</td>
<td>5.08</td>
<td>2.00</td>
<td>5.08</td>
<td>0.24</td>
<td>7.07</td>
</tr>
<tr>
<td>50</td>
<td>3.5</td>
<td>678</td>
<td>5.045</td>
<td>1.98</td>
<td>5.04</td>
<td>0.28</td>
<td>8.12</td>
</tr>
<tr>
<td>55</td>
<td>3.3</td>
<td>756</td>
<td>5.012</td>
<td>1.97</td>
<td>5.01</td>
<td>0.31</td>
<td>9.05</td>
</tr>
<tr>
<td>60</td>
<td>3.1</td>
<td>843</td>
<td>4.962</td>
<td>1.95</td>
<td>4.96</td>
<td>0.37</td>
<td>10.09</td>
</tr>
</tbody>
</table>
Using the data in table 5, a plot of levitation height vs. velocity was made, shown in figure 12.

Figure 14: Plot of Levitation Height vs. Velocity
Discussion of Results:

The experimental results found in lab can be compared to the Matlab simulation and theoretical calculations to see if they match up.

The data gathered using the displacement transducer can be compared to the Matlab simulation. It can be seen that there was a simulated levitation height of about 0.85 cm at a speed of 10 m/s. Experimental results showed a levitation height of 0.37 cm at a speed of 10.09 m/s. When the trial was run with the displacement sensor, the Halbach array device started the experiment with a distance of about 0.5 cm from the track. If this is taken into account, the 0.37 cm must be added to the 0.5 cm. This would have resulted in a levitation height of almost exactly 0.85 cm, as the simulation results predicted.

Another set of parameters that can be compared are the simulated and experimental lift and drag forces. The lift forces were measured with the force sensor. These values can be compared to the simulated lift forces from the Matlab GUI. Looking at the Matlab simulation, it can be seen that the levitation force at 10 m/s was 20.6 Newtons. This force does not take into account the forces acting in the downward direction due to the mass of the train. Therefore, the actual force would be 20.6 - (9.81 * 0.443), which is 16.25 Newtons. The experiment results show a force of 15 Newtons at 9 m/s. If these results are extrapolated out, it can be seen that the experimental levitation force matches the simulated levitation force.

Looking at the calculated levitation and drag forces on page 8 and the Matlab GUI, the values for levitation force should match, but they do not. The GUI gave a levitation force of 20.6 N and a drag force of 7.37 N, whereas the equation gave a levitation force of 21.0 N and a drag force of 2.4 N. This is believed to be due to a parameter, called levs, that Paul Friend uses as a scale factor in his calculations. This scale factor is to account for the fringe fields of the Halbach array creating forces that are not equal to the forces at the center of the array. This scale factor was not taken into account in calculations, which would lead to a different value for both the levitation force and drag force. Furthermore, the calculations contain many variables and calculations of other parameters, so human error in calculation could have played a role in the different values. In addition, it is not known whether or not the levitation height was taken into account when the force was calculated in the Matlab GUI. Therefore, the height of 5 mm that was used for calculations would be incorrect if the GUI used 5mm plus the levitation height.

The experimental and simulated drag forces can also be compared. The drag forces can be calculated in the following manner:

First, the power needs to be computed. The current measurements were taken in order to calculate this value.

\[ P = I^*V^*\eta \]  \hspace{1cm} (15)

Where \( \eta \) is the efficiency of the motor.
Power is also shown in the follow equation:

\[ P = T^*\omega \]  \hspace{1cm} \text{(16)}

Where \( T \) is the torque and \( \omega \) is the speed in rad/sec.

Equating equations (15) and (16), the motor torque can be found.

\[ T = V^*I^*\eta/\omega \]  \hspace{1cm} \text{(17)}

Using the equation for torque,

\[ T = F^*l \]  \hspace{1cm} \text{(18)}

Where \( F \) is the force in newtons and \( l \) is the lever arm in meters

The force can be solved for. This force will be equal to the drag force.

The results of drag force calculations, using the data from the trial with displacement data, are shown in table 6.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>140</td>
<td>1.68</td>
<td>36.00</td>
<td>14.66</td>
<td>2.46</td>
<td>10.74</td>
</tr>
<tr>
<td>25</td>
<td>3.7</td>
<td>277</td>
<td>3.32</td>
<td>74.00</td>
<td>29.01</td>
<td>2.55</td>
<td>11.16</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>347</td>
<td>4.15</td>
<td>96.00</td>
<td>36.34</td>
<td>2.64</td>
<td>11.56</td>
</tr>
<tr>
<td>35</td>
<td>4.4</td>
<td>417</td>
<td>4.99</td>
<td>123.20</td>
<td>43.67</td>
<td>2.82</td>
<td>12.34</td>
</tr>
<tr>
<td>40</td>
<td>3.9</td>
<td>505</td>
<td>6.04</td>
<td>124.80</td>
<td>52.88</td>
<td>2.36</td>
<td>10.32</td>
</tr>
<tr>
<td>45</td>
<td>3.6</td>
<td>591</td>
<td>7.07</td>
<td>129.60</td>
<td>61.89</td>
<td>2.09</td>
<td>9.16</td>
</tr>
<tr>
<td>50</td>
<td>3.5</td>
<td>678</td>
<td>8.12</td>
<td>140.00</td>
<td>71.00</td>
<td>1.97</td>
<td>8.63</td>
</tr>
<tr>
<td>55</td>
<td>3.3</td>
<td>756</td>
<td>9.05</td>
<td>145.20</td>
<td>79.17</td>
<td>1.83</td>
<td>8.02</td>
</tr>
<tr>
<td>60</td>
<td>3.1</td>
<td>843</td>
<td>10.09</td>
<td>148.80</td>
<td>88.28</td>
<td>1.69</td>
<td>7.37</td>
</tr>
</tbody>
</table>

A data sheet for the Dayton motor could not be found, therefore, an efficiency of 80% was assumed for drag force calculations. The drag force at different heights can be compared to the theoretical drag forces found in the Matlab simulation and in the design.
The simulated drag forces were 5.4 Newtons, which match closely to the experimental results of 7.37 Newtons at 10 m/s. There are many sources of error that could have contributed to the difference in simulation and results. One of these is the values for L and R could have been different on the actual track than what the theoretical values were. If there was extra track, a piece of track could have been cut out and R and L could have been measured. This would have made a more accurate simulation.

Another parameter that could be looked into is the lift to drag ratio. This would use the drag force calculations in table 6. However, this ratio cannot be determined accurately from the data collected. In order to compute lift/drag, levitation forces would need to be measured at specific heights, corresponding to the drag force data at specific levitation heights. The current data that has been collected would give inaccurate ratios since the forces were measured with a stationary levitation device.
Equipment List:

- 9” radius polyethylene wheel, with a width of 2”
- 57”x2”x1/4” copper sheet of thin conducting strips
- 125 - 6mm cube neodymium magnets
- Balsa wood structure to house the 5x25 Halbach array
- Bearings and coupler for motor shaft
- Heat shrink for gluing of magnets
- Metal and hardware for motor stand
- Dayton permanent magnet DC motor
- Digital Force Gauge Model: 475040
- Displacement Transducer Model: MLT002N3000B5C

Table 7: Purchased Equipment

<table>
<thead>
<tr>
<th>Item Description</th>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>6mm cubed N38 Magnets</td>
<td>125</td>
<td>$0.38/magnet</td>
</tr>
<tr>
<td>Copper Sheet- 1/8”x4”x6’</td>
<td>1</td>
<td>$158.89</td>
</tr>
<tr>
<td>LDPE Block - 2”x24”x24”</td>
<td>1</td>
<td>$238.78</td>
</tr>
<tr>
<td>Track Fabrication</td>
<td>1</td>
<td>$350.00</td>
</tr>
<tr>
<td>Wheel Fabrication</td>
<td>1</td>
<td>$150.00</td>
</tr>
<tr>
<td>Bearing</td>
<td>2</td>
<td>$39.68</td>
</tr>
<tr>
<td>Motor Coupler</td>
<td>1</td>
<td>$16.42</td>
</tr>
<tr>
<td>Epoxy</td>
<td>1</td>
<td>$15.00</td>
</tr>
<tr>
<td>Heat Shrink Tube</td>
<td>3 foot</td>
<td>$4.65/ft</td>
</tr>
</tbody>
</table>
Acknowledgements:

Dr. Winfred Anakwa – assistance with understanding of theory and help with fabrication

Mr. Steven Gutschlag – assistance with understanding of theory, fabrication, and troubleshooting

Mr. Joe Richey and Tri-City Machining – assistance with design and machining of the plastic wheel and copper track

Mr. Darren DeDecker and Caterpillar Inc. – assistance with machining of small amount off of the wheel

Mrs. Sue DeDecker – assistance with gluing of magnets

Mr. Dave Miller – assistance with parts fabrication

Mr. Harrison Cohen – assistance with formulation of ProE and AutoCad drawings for parts machining

Dr. Sam Gurol – previous research and assistance with understanding of theory

Mr. Paul Friend – previous research

Mr. Glenn Zomcheck – previous research
Applicable Patents:

- Coffey; Howard T.
  Propulsion and stabilization for magnetically levitated vehicles
  U.S. Patent 5,222,436
  June 29, 2003

- Coffey; Howard T.
  Magnetic Levitation configuration incorporating levitation, guidance and linear synchronous motor
  U.S. Patent 5,253,592
  October 19, 1993

- Levi; Enrico; Zabar; Zivan
  Air cored, linear induction motor for magnetically levitated systems
  U.S. Patent 5,270,593
  November 10, 1992

- Lamb; Karl J.; Merrill; Toby; Gossage; Scott D.; Sparks; Michael T.; Barrett; Michael S.
  U.S. Patent 6,510,799
  January 28, 2003

- Richard F. Post
  Magnetic Levitation System for Moving Objects
  U.S. Patent 5,722,326
  March 3, 1998

- Richard F. Post
  Inductrack Magnet Configuration
  U.S. Patent 6,633,217 B2
  October 14, 2003

- Richard F. Post
  Inductrack Configuration
  U.S. Patent 629,503 B2
  October 7, 2003

- Richard F. Post
  Laminated Track Design for Inductrack Maglev System
  U.S. Patent Pending US 2003/0112105 A1
  June 19, 2003
Bibliography:


[6] Sam Gurol, E-mail (Private Conversation)