Exp - 9 Measurements of a Mixer

Handout used for Fall 2004

1. Handout on Detectors & Mixers
Objective: To understand the operational characteristics and applications of a mixer.

Equipment:
- Network Analyzer (HP 8722C)
- Spectrum Analyzer (HP 8593E)
- Signal Generator (HP E4433B)
- HP8620C Sweep Oscillator with 86240B RF Plug In (2.0 – 8.4 GHz)

Components:
- DC Block (using Bias-T - ZFBT-6GW-FT)
- Mixer (ZFM-4212)
- Band pass filter (SBP-70)
- Band pass filter (fabricated, 2.4 GHz Side Coupled Filter)
- 2 SMA(M)-SMA(M) and 3 N(M)-SMA(F) Adapters
- 50 ohm termination load
- 2.5 ft cable for signal generator & sweep oscillator (Storm B/N 8650)
- 2 ft cable for spectrum analyzer (Precision Tube Co. OBV02, A021349201-024)

Prelab:
1. Study the handout on “Detectors and Mixers”.
2. With the help of schematic diagrams, briefly explain how a mixer can be used for a) Down conversion, b) Up conversion.
3. What are “image” frequencies? Why should they be avoided? How can we avoid them?
4. Define “Conversion loss” for a passive mixer.
5. By considering the linear, quadratic and cubic terms in the equation relating input and output (p.574 in handout) in a mixer, list the frequency components present in the output of a mixer.

Procedures:
I. Perform a full 2-port calibration in the frequency range 50 to 110 MHz on the network analyzer. Use number of points as 201. Measure/record $S_{21}$ and $S_{11}$ of the band pass filter SBP-70 using the dual channel display. Also, display the 3dB-bandwidth and center frequency using the bandwidth statistics functionality.

II. Perform a full 2-port calibration in the frequency range 50 MHz to 6 GHz on the network analyzer. Measure/record $S_{21}$ and observe/record the behavior of the band pass filter at higher frequencies.

III. MIXER AS A DOWN CONVERTER:
The intermediate frequency (f_{IF}) is selected to be 70 MHz.

a) Set the signal generator to 2.4 GHz at 0dBm – this will be the RF signal. Note that the actual signal power seen on the spectrum analyzer will be close to –1dBm due to cable and connector losses – record
this loss difference (to be used in a later section). Observe/record the generated signal along with its harmonics and spurious frequencies if any. To do this, use the Band Lock option in the frequency menu of the spectrum analyzer. Choose Frequency, Band Lock, Band Lock ON. Then observe the different frequency bands available using 300 KHz for the bandwidth resolution in each frequency band – adjust the start and stop frequency for your observation.

b) Set the sweep oscillator to 2.47 GHz at 1dBm – this will be the LO frequency. The sweep oscillator settings are inaccurate; therefore, you must set the signal according to what you see on the spectrum analyzer. Observe/record the generated signal along with its harmonics and spurious frequencies if any. To do this, use the Band Lock option in the frequency menu of the spectrum analyzer. Choose Frequency, Band Lock, Band Lock ON. Then observe the different frequency bands available using 300 KHz for the bandwidth resolution in each frequency band.

c) Connect the mixer as a down converter as shown in Fig.1

**NOTE:** Do not connect the band pass filter in this step. The band pass filter is used in a subsequent setup.

![Diagram](image1)

**Fig.1 Mixer as a Down Converter**

d) Record the signal levels present at various frequencies for different RF signal levels using Table 1. Observe the intermodulation products obtained and note your comments referring to parts (a) and (b). Use the Band Lock option in the frequency menu of the spectrum analyzer as explained in parts (a) and (b). Note: the Band Lock option should be used throughout the entire experiment.

**Table 1**

<table>
<thead>
<tr>
<th>$P_{RF}$ (dBm)</th>
<th>$f_{RF}$ (dBm)</th>
<th>$f_{LO}$ (dBm)</th>
<th>$2f_{RF}-f_{LO}$ (dBm)</th>
<th>$2f_{LO}-f_{RF}$ (dBm)</th>
<th>$f_{LO}-f_{RF}$ (dBm)</th>
<th>$f_{LO}+f_{RF}$ (dBm)</th>
<th>$2f_{LO}+f_{RF}$ (dBm)</th>
<th>$f_{LO}+2f_{RF}$ (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</table>

e) LO – RF Isolation. Disconnect the spectrum analyzer from the IF port on the mixer. Terminate the IF port with 50Ω. Disconnect the sweep oscillator from the LO port and connect the spectrum analyzer instead, using the cable Precision Tube Co. OBV02, A021349201-024. Set the signal generator power level back to 0dBm, and record the power level observed at $f_{RF}$ at the LO port. Calculate the isolation and refer to the mixer specifications to check your answer. **Note:** take into consideration the loss of the
cable and connectors obtained in part (a) as well as the loss of the cable, Precision Tube Co. OBV02, A021349201-024, which is 0.264 dBm.

f) LO – IF Isolation. From the data obtained in table 1, note the power level obtained at $f_{LO}$ at the IF port, and calculate the isolation. Refer to the mixer specifications to check your answer. Note: take into account the loss of the cable, Storm 90-010-036 (connected from the sweep oscillator to the LO port), which is 0.244 dBm, as well as the loss of the cable, A021349201-024, which is 0.264 dBm.

Reconnect the mixer as a down converter including the band pass filter (SBP-70) as shown in Fig.1. Repeat step (d) using Table 2 to record your data. Observe the intermodulation products obtained and note your comments referring to parts I and II as well as parts (a) and (b). Note: use 30 kHz for the resolution bandwidth.

TABLE 2

<table>
<thead>
<tr>
<th>$P_{RF}$ (dBm)</th>
<th>$f_{RF}$ (dBm)</th>
<th>$f_{LO}$ (dBm)</th>
<th>$2f_{LO}-f_{RF}$ (dBm)</th>
<th>$2f_{LO}-f_{RF}$ (dBm)</th>
<th>$f_{LO}+f_{RF}$ (dBm)</th>
<th>$f_{LO}+f_{RF}$ (dBm)</th>
<th>$f_{LO}+2f_{RF}$ (dBm)</th>
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</tbody>
</table>

h) Set $f_{RF}$ (signal generator) to the upper image frequency ($f_{IM}=f_{RF}+f_{IM}$) and keep the power level at 0 dBm. Record the frequency components (those given in the tables) observed on the spectrum analyzer along with their corresponding power levels. NOTE: use 30 kHz for the resolution bandwidth.

i) Set $f_{LO}$ (sweep oscillator) to 2.4 GHz keeping the power level at 1 dBm. Set $f_{RF}$ (signal generator) to the lower image frequency ($f_{IM}=f_{RF}-f_{IM}$) and keep the power level at 0 dBm. Record the frequency components (those given in the tables) observed on the spectrum analyzer along with their corresponding power levels. NOTE: use 30 kHz for the resolution bandwidth and use Band Lock position.

IV. MIXER AS AN UP-CONVERTER:

a) Set the signal generator to 100 MHz at 0 dBm – this will be the IF signal.

b) Set the sweep oscillator to 2.24 GHz at 1 dBm – this will be the LO frequency. The sweep oscillator settings are inaccurate; therefore, you must set the signal according to what you see on the spectrum analyzer.

c) Connect the mixer as an up-converter as shown in Fig.2.

**NOTE:** Do not connect the band pass filter in this step. It is used in a subsequent setup.

**Fig.2 Mixer as an Up-converter**
d) Record the signal levels present at the various frequencies using Table 3. Note: use the Band Lock option with a resolution bandwidth of 300 kHz in each frequency band.

<table>
<thead>
<tr>
<th>$P_{IF}$ (dBm)</th>
<th>$f_{IF}$ (dBm)</th>
<th>$f_{LO}$ (dBm)</th>
<th>$f_{LO}-2f_{IF}$ (dBm)</th>
<th>$2f_{LO}-f_{IF}$ (dBm)</th>
<th>$f_{LO}-f_{IF}$ (dBm)</th>
<th>$f_{LO}+f_{IF}$ (dBm)</th>
<th>$2f_{LO}+f_{IF}$ (dBm)</th>
<th>$f_{LO}+2f_{IF}$ (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>


e) Include the band pass filter (2.34 GHz) to the up-converter setup as shown in Fig.2. The band pass filter response is attached. Repeat step (d) using Table 4 to record your data.

<table>
<thead>
<tr>
<th>$P_{IF}$ (dBm)</th>
<th>$f_{IF}$ (dBm)</th>
<th>$f_{LO}$ (dBm)</th>
<th>$f_{LO}-2f_{IF}$ (dBm)</th>
<th>$2f_{LO}-f_{IF}$ (dBm)</th>
<th>$f_{LO}-f_{IF}$ (dBm)</th>
<th>$f_{LO}+f_{IF}$ (dBm)</th>
<th>$2f_{LO}+f_{IF}$ (dBm)</th>
<th>$f_{LO}+2f_{IF}$ (dBm)</th>
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<tbody>
<tr>
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</tbody>
</table>

Report:
1. Calculate the Conversion loss (dB) of the mixer for different RF power levels in Table 1 and Table 2.
2. What type of filter response (bandwidth and skirt response) do you recommend at the outputs of the down and up converters (Fig.1 and Fig.2) to get the "ideal outputs"?
3. Referring to the handout on mixers, identify various components in Table 1 and Table 2 as IF component, LO – RF isolation, LO – IF isolation, Sum Component, and Second order intermodulation products.
4. Referring to Table 1, comment on the power level variations of various frequency components with the variation in $P_{RF}$ (dBm). Which RF power level ($P_{RF}$) do you recommend for optimum performance of the mixer?
### Ultra-Rel Mixers

**Specifications**

- **+7 dBm LO, up to +1 dBm RF**

<table>
<thead>
<tr>
<th>Model</th>
<th>Frequency</th>
<th>Conversion Loss (dB)</th>
<th>Loop Isolation (dB)</th>
<th>Loop Isolation (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZAM-25</td>
<td>10-1000 DC-1000</td>
<td>5.74</td>
<td>7.0</td>
<td>8.5</td>
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<tr>
<td>ZAM-1800</td>
<td>2000-4000 DC-1000</td>
<td>6.65</td>
<td>4.0</td>
<td>3.5</td>
</tr>
</tbody>
</table>

- **NSN Guide**

  - **NSN NO.**
    - ZAM-1800DC: 5985-00-285-7750
    - ZAM-48: 5985-01-127-0376
    - ZAM-68: 5985-01-346-7865
    - ZAM-2: 5985-01-235-1684
    - ZAM-16: 5985-01-235-3037
    - ZAM-21: 4135-01-230-4782
    - ZAM-3: 5985-01-057-8523
    - ZAM-3 (MD): 5985-01-264-1782
    - ZAM-38: 5985-01-381-0299
    - ZAM-31-3: 6625-01-232-1622
    - ZAM-1: 5985-00-667-7010
    - ZAM-9: 6625-01-021-1018
    - ZAM-9: 6625-01-381-0299
    - ZAM-31-4: 6625-01-169-0156
    - ZAM-31-4 (NC): 5995-01-384-7451

- **Coaxial Connections**

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<th>PORT</th>
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<th>3</th>
<th>4</th>
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**In Stock... Immediate Delivery**

For Custom Versions Of Standard Models Consult Our Applications Dept.
Filters bandpass
50 ohm
10.7 MHz to 70 MHz
Center Frequency

Case style selection
(outline drawings see Table of Contents)

Constant Impedance

<table>
<thead>
<tr>
<th>MODEL NO.</th>
<th>CENTER FREQUENCY</th>
<th>PASS BAND, MHz</th>
<th>STOP BANDS</th>
<th>VSWR, 1.3:1, Typ.</th>
<th>PRICE</th>
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<td>20-214</td>
<td></td>
<td>21-4</td>
<td>4.9 &amp; 85</td>
<td>1.3 &amp; 100</td>
<td>14.95</td>
</tr>
<tr>
<td>20-30</td>
<td></td>
<td>23-35</td>
<td>7 &amp; 120</td>
<td>1.9 &amp; 260</td>
<td>14.95</td>
</tr>
<tr>
<td>20-40</td>
<td></td>
<td>35-45</td>
<td>10 &amp; 160</td>
<td>2.6 &amp; 330</td>
<td>14.95</td>
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<tr>
<td>20-50</td>
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<td>41-54</td>
<td>11.5 &amp; 200</td>
<td>3.1 &amp; 330</td>
<td>14.95</td>
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<tr>
<td>20-60</td>
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<td>50-70</td>
<td>16 &amp; 240</td>
<td>3.6 &amp; 400</td>
<td>14.95</td>
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<tr>
<td>20-70</td>
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<td>56-82</td>
<td>16 &amp; 280</td>
<td>4.4 &amp; 490</td>
<td>14.95</td>
</tr>
</tbody>
</table>

*Add prefix letter P, B, N or S to _F where applicable (see note 4)

Elliptic Response

<table>
<thead>
<tr>
<th>MODEL NO.</th>
<th>CENTER FREQUENCY</th>
<th>PASS BAND, LL. 50 DB MAX. (MHz)</th>
<th>3 DB BANDWIDTH TYPICAL (MHz)</th>
<th>STOP BANDS</th>
<th>PASS BAND VSWR MAX.</th>
<th>STOP BAND VSWR TYPICAL</th>
<th>PRICE</th>
</tr>
</thead>
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<tr>
<td>20P-10.7</td>
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<td>9.0-11.5</td>
<td>9.0-12.7</td>
<td>1.2 &amp; 10</td>
<td>0.4 &amp; 1000</td>
<td>1.7:1</td>
<td>16.1</td>
</tr>
<tr>
<td>20P-21.4</td>
<td></td>
<td>9.2-23.6</td>
<td>11-25</td>
<td>10.5 &amp; 29</td>
<td>1.0 &amp; 80-1000</td>
<td>1.7:1</td>
<td>16.1</td>
</tr>
<tr>
<td>20P-30</td>
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<td>27.0-35.0</td>
<td>25-35</td>
<td>22 &amp; 40</td>
<td>2.0 &amp; 90-1000</td>
<td>1.7:1</td>
<td>16.1</td>
</tr>
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<td>20P-40</td>
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<td>50.0-67.0</td>
<td>49.0-105.0</td>
<td>44 &amp; 79</td>
<td>4.6 &amp; 190-1000</td>
<td>1.7:1</td>
<td>16.1</td>
</tr>
<tr>
<td>20P-50</td>
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<td>65.0-77.0</td>
<td>60.0-82.0</td>
<td>51 &amp; 94</td>
<td>6.0 &amp; 190-1000</td>
<td>1.7:1</td>
<td>16.1</td>
</tr>
</tbody>
</table>

*Add prefix letter P, B, N or S to _BP where applicable (see note 5)

Pin Connections

[Diagram showing pin connections]

Mini-Circuits
noise figure of the transmission line-amplifier cascade. What would be the noise figure if the amplifier were placed at the antenna, eliminating the transmission line? Assume all components are at an ambient temperature of \( T = 300 \text{ K} \).

**Solution**

The loss factor of the coaxial line is \( L = 10^{2/10} = 1.58 \), so from (10.16) the noise figure of the line is

\[
F_L = 1 + (L - 1) \frac{T}{T_0} = 1 + (1.58 - 1) \frac{300}{290} = 1.60 = 2.04 \text{ dB}.
\]

From (10.11), the noise figure of the amplifier is

\[
F_a = 1 + \frac{T_a}{T_0} = 1 + \frac{150}{290} = 1.52 = 1.81 \text{ dB}.
\]

Then (10.21) gives the noise figure of the cascade as

\[
F_{cas} = F_L + \frac{1}{G_L} (F_a - 1) = 1.60 + 1.58(1.52 - 1) = 2.42 = 3.84 \text{ dB},
\]

since \( 1/G_L = L = 1.58 \) for the coaxial line. Without the transmission line, the noise figure would be that of the amplifier itself, or 1.81 dB. So we see that the effect of the lossy feedline reduces the noise figure of the system by about 2 dB—a substantial amount. Sometimes such a line cannot be avoided in the front end of a receiver. Its effect, however, will be deleterious, because not only does the line itself add noise but, since its gain is less than unity, it increases the effect of the noise of the next stage.

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**10.2 Detectors and Mixers**

Detectors and mixers use a nonlinear device to achieve *frequency conversion* of an input signal [1]. Microwave diodes are most commonly used as the nonlinear element, but transistors can also be used. Figure 10.10 illustrates the three basic frequency conversion functions of rectification, detection, and mixing. We will first discuss the nonlinear voltage-current characteristics of a diode, and then use a small-signal analysis to describe the operation of various circuits that perform these functions.

**Diode Rectifiers and Detectors**

A diode is basically a nonlinear resistor, with a DC *V-I* characteristic that can be expressed as

\[
I(V) = I_s (e^{\alpha V} - 1),
\]

where \( \alpha = q/nkT \), and \( q \) is the charge of an electron, \( k \) is Boltzmann’s constant, \( T \) is temperature, \( n \) is the ideality factor, and \( I_s \) is the saturation current [4], [5]. Typically,
\( I_a \) is between \( 10^{-6} \) and \( 10^{-15} \) A, and \( \alpha = q/nkT \) is approximately \( 1/(25 \text{ mV}) \) for \( T = 290 \text{ K} \). The ideality factor, \( n \), depends on the structure of the diode itself, and can vary from 1.2 for Schottky barrier diodes to about 2.0 for point-contact silicon diodes. Figure 10.11 shows a typical diode \( V-I \) characteristic. Now let the diode voltage be

\[
V = V_0 + v, \tag{10.25}
\]

where \( V_0 \) is a DC bias voltage and \( v \) is a small AC signal voltage. Then (10.24) can be expanded in a Taylor series about \( V_0 \) as follows:

\[
I(V) = I_0 + v \frac{dI}{dV} \bigg|_{V_0} + \frac{1}{2} v^2 \frac{d^2I}{dV^2} \bigg|_{V_0} + \cdots, \tag{10.26}
\]
where \( I_0 = I(V_0) \) is the DC bias current. The first derivative can be evaluated as

\[
\left. \frac{dI}{dV} \right|_{V_0} = \alpha I_s e^{\alpha V_0} = \alpha (I_0 + I_s) = G_d = \frac{1}{R_j},
\]

which defines \( R_j \), the junction resistance of the diode, and \( G_d = 1/R_j \), which is called the dynamic conductance of the diode. The second derivative is

\[
\left. \frac{d^2 I}{dV^2} \right|_{V_0} = \frac{dG_d}{dV} \left|_{V_0} = \alpha^2 I_s e^{\alpha V_0} = \alpha^2 (I_0 + I_s) = \alpha G_d = G'_d. \right.
\]

Then (10.26) can be rewritten as the sum of the DC bias current, \( I_0 \), and an AC current, \( i \):

\[
I(V) = I_0 + i = I_0 + vG_d + \frac{v^2}{2} G'_d + \cdots.
\]

The three-term approximation for the diode current in (10.29) is called the small-signal approximation, and will be adequate for most of our purposes.

The small-signal approximation is based on the DC voltage-current relationship of (10.24), and shows that the equivalent circuit of a diode will involve a nonlinear resistance. In practice, however, the AC characteristics of a diode also involve reactive effects due to the structure and packaging of the diode. A typical equivalent circuit for a diode is shown in Figure 10.12. The leads and contacts of the diode package lead to a series inductance, \( L_p \), and shunt capacitance, \( C_p \). The series resistor, \( R_s \), accounts for

![Equation Diagram](image-url)
contact and current-spreading resistance. $C_j$ and $R_j$ are the junction capacitance and resistance, and are bias-dependent.

**EXAMPLE 10.4 Diode Package Effects**

A diode in an axial-lead package has the following equivalent circuit parameters: $C_p = 0.10$ pF, $L_p = 2.0$ nH, $C_j = 0.15$ pF, $R_s = 10$ $\Omega$, and $I_s = 0.1$ $\mu$A. Calculate and plot the impedance of this diode from 4 to 14 GHz, for a bias current $I_0 = 0$ and $I_0 = 60$ $\mu$A. Ignore the change in $C_j$ with bias, and assume $\alpha = 1/(25$ mV).

**Solution**

From (10.27) the junction resistance for the two bias states is

- for $I_0 = 0$, $R_j = \frac{1}{\alpha(I_0 + I_s)} = \frac{25$ mV}{0.1$ \mu$A} = $2.5 \times 10^5$ $\Omega$,
- for $I_0 = 60$ $\mu$A, $R_j = \frac{1}{\alpha(I_0 + I_s)} = \frac{25$ mV}{(60 + 0.1)$ \mu$A} = 417 $\Omega$.

Then the input impedance can be calculated from the equivalent circuit of Figure 10.12; the result is plotted versus frequency on a 50$\Omega$ Smith chart in Figure 10.13.

**FIGURE 10.13** Impedance of the diode of Example 10.4 for two bias states, from 4 to 14 GHz.
In a rectifier application, a diode is used to convert a fraction of an RF input signal to DC power. Rectification is a very common function, and is used for power monitors, automatic gain control circuits, and signal strength indicators. If the diode voltage consists of a DC bias voltage and a small-signal RF voltage,

\[ V = V_0 + v_0 \cos \omega_0 t, \]

then (10.29) shows that the diode current will be

\[ I = I_0 + v_0 G_d \cos \omega_0 t + \frac{v_0^2}{2} G'_d \cos^2 \omega_0 t \]

\[ = I_0 + \frac{v_0^2}{2} G'_d + v_0 G_d \cos \omega_0 t + \frac{v_0^2}{4} G'_d \cos 2\omega_0 t. \]

\[ I_0 \text{ is the bias current and } \frac{v_0^2 G'_d}{4} \text{ is the DC rectified current. The output also contains AC signals of frequency } \omega_0 \text{, and } 2\omega_0 \text{ (and higher-order harmonics), which are usually filtered out with a simple low-pass filter. A current sensitivity, } \beta_i, \text{ can be defined as a measure of the change in DC output current for a given input RF power. From (10.29) the RF input power is } \frac{v_0^2 G_d}{2} \text{ (using only the first term), while (10.31) shows the change in DC current is } \frac{v_0^2 G'_d}{4} \text{. The current sensitivity is then} \]

\[ \beta_i = \frac{\Delta I_{dc}}{P_m} = \frac{G'_d}{2G_d} \text{ A/W}. \]

An open-circuit voltage sensitivity, \( \beta_v \), can be defined in terms of the voltage drop across the junction resistance when the diode is open-circuited. Thus,

\[ \beta_v = \beta_i R_j. \]

Typical values for the voltage sensitivity of a diode range from 400 to 1500 mV/mW.

In a detector application the nonlinearity of a diode is used to demodulate an amplitude modulated RF carrier. For this case, the diode voltage can be expressed as

\[ v(t) = v_0(1 + m \cos \omega_m t) \cos \omega_0 t, \]

where \( \omega_m \) is the modulation frequency, \( \omega_0 \) is the RF carrier frequency (\( \omega_0 >> \omega_m \)), and \( m \) is defined as the modulation index (0 \( \leq m \leq 1 \)). Using (10.34) in (10.29) gives the diode current:

\[ i(t) = v_0 G_d (1 + m \cos \omega_m t) \cos \omega_0 t + \frac{v_0^2}{2} G'_d (1 + m \cos \omega_m t)^2 \cos^2 \omega_0 t \]

\[ = v_0 G_d \left[ \cos \omega_0 t + \frac{m}{2} \cos(\omega_0 + \omega_m) t + \frac{m}{2} \cos(\omega_0 - \omega_m) t \right] \]

\[ + \frac{v_0^2}{4} G'_d \left[ 1 + \frac{m^2}{2} + 2m \cos \omega_m t + \frac{m^2}{2} \cos 2\omega_m t + \cos 2\omega_0 t \right. \]

\[ + m \cos(2\omega_0 + \omega_m) t + m \cos(2\omega_0 - \omega_m) t + \frac{m^2}{2} \cos 2\omega_0 t \]

\[ + \frac{m^2}{4} \cos 2(\omega_0 + \omega_m) t + \frac{m^2}{4} \cos 2(\omega_0 - \omega_m) t \]
The frequency spectrum of this output is shown in Figure 10.14. The output current terms which are linear in the diode voltage (terms multiplying $v_0 G_d$) have frequencies of $\omega_0$ and $\omega_0 \pm \omega_m$, while the terms that are proportional to the square of the diode voltage (terms multiplying $v_0^2 G_d^2/2$) include the frequencies and relative amplitudes listed in Table 10.1.

The desired demodulated output of frequency $\omega_m$ is easily separated from the undesired components with a low-pass filter. Observe that the amplitude of this current is $mv_0^2 G_d^2/2$, which is proportional to the power of the input signal. This square-law behavior is the usual operating condition for detector diodes, but can be obtained only over a restricted range of input powers. If the input power is too large, small-signal conditions will not apply, and the output will become saturated and approach a linear, and then a constant, $i$ versus $P$ characteristic. At very low signal levels the input signal will be lost in the noise floor of the device. Figure 10.15 shows the typical $v_{out}$ versus $P_n$ characteristic, where the output voltage can be considered as the voltage drop across a resistor in series with the diode. Square-law operation is particularly important for applications where power levels are inferred from detector voltage, as in SWR indicators and signal level indicators. Detectors may be DC biased to an operating point that provides the best sensitivity.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Relative Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1 + m^2/2$</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>2$m$</td>
</tr>
<tr>
<td>2$\omega_m$</td>
<td>$m^2/2$</td>
</tr>
<tr>
<td>2$\omega_0$</td>
<td>1 + $m^2/2$</td>
</tr>
<tr>
<td>2$\omega_0 \pm \omega_m$</td>
<td>$m$</td>
</tr>
<tr>
<td>2($\omega_0 \pm \omega_m$)</td>
<td>$m^2/4$</td>
</tr>
</tbody>
</table>
Single-Ended Mixer

A mixer uses the nonlinearity of a diode to generate an output spectrum consisting of the sum and difference frequencies of two input signals. In a receiver application, a low-level RF signal and an RF local oscillator (LO) signal are mixed together to produce an intermediate frequency (IF), \( f_{IF} = f_{RF} - f_{LO} \), and a much higher frequency, \( f_{RF} + f_{LO} \), which is filtered out. See Figure 10.16a. The IF signal usually has a frequency between 10 and 100 MHz, and can be amplified with a low-noise amplifier. This is called a heterodyne receiver, and is useful because it has much better sensitivity and noise characteristics (using an IF amplifier minimizes \( 1/f \) noise) than the direct detection scheme discussed in the previous section. A heterodyne system also has the advantage of being able to tune over a band by simply changing the LO frequency, without the need for a high-gain, wideband RF amplifier.

As shown in Figure 10.16b, a mixer can also be used in a transmitter to offset the frequency of an RF signal by an amount equal to \( f_{IF} \). This is a convenient technique, as it allows the use of identical local oscillators in the transmitter and receiver; a single oscillator may serve this purpose in a radar or transceiver system.

There are several types of mixer circuits, but the simplest is the single-ended mixer; single-ended mixers often are used as part of more sophisticated mixers. A typical single-ended mixer circuit is shown in Figure 10.17, where an RF signal,

\[ v_{RF}(t) = v_r \cos \omega_r t, \tag{10.36} \]

is combined with an LO signal,

\[ v_{LO}(t) = v_0 \cos \omega_0 t, \tag{10.37} \]

and fed into a diode. The combiner may be a simple \( T \)-junction combiner, or a directional coupler. An RF matching circuit may precede the diode, and the diode may be biased through chokes that allow DC to pass while blocking RF. From (10.29), the diode current
will consist of a constant DC bias term, and RF and LO signals of frequencies \( \omega_r \) and \( \omega_0 \), due to the term which is linear in \( v \). The \( v^2 \) term will give rise to the following output current:

\[
\begin{align*}
    i &= \frac{G_d^*}{2} (v_r \cos \omega_r t + v_0 \cos \omega_0 t)^2 \\
    &= \frac{G_d^*}{2} (v_r^2 \cos^2 \omega_r t + 2v_r v_0 \cos \omega_r t \cos \omega_0 t + v_0^2 \cos^2 \omega_0 t)
\end{align*}
\]
\[ G(\xi) = \frac{G^2}{4} \{ v_r^2 + v_0^2 + v_0^2 \cos 2\omega_0 t + v_0^2 \cos 2\omega_0 t + 2v_r v_0 \cos (\omega_r - \omega_0) t + 2v_r v_0 \cos (\omega_r + \omega_0) t \}. \]  

10.38

The DC terms can be ignored, and the \( 2\omega_r \) and \( 2\omega_0 \) terms will be filtered out. The most important terms are those of frequency \( \omega_r \pm \omega_0 \).

For a receiver or down-converter, the \( \omega_r - \omega_0 \) term will become the IF signal. Note that, for a given local oscillator frequency, there will be two RF frequencies that will mix down to the same IF frequency. If the RF frequency is \( \omega_r = \omega_0 + \omega_l \), then the output frequencies of the mixer will be \( \omega_r \pm \omega_0 = 2\omega_0 + \omega_l \), and \( \omega_l \); if the RF frequency is \( \omega_r = \omega_0 - \omega_l \), the mixer output frequencies will be \( \omega_r \pm \omega_0 = 2\omega_0 - \omega_l \), and \( -\omega_l \). This latter output is called the image response of the mixer, and is indistinguishable from the direct response. It can be eliminated by RF filtering at the input of the mixer, but this is difficult because the desired RF frequency \( (\omega_0 + \omega_l) \) is relatively close to the spurious image frequency \( (\omega_0 - \omega_l) \), since generally \( \omega_l \ll \omega_0 \). Another way to eliminate the image response is by using an image rejection mixer.

In an up-converter, or modulation, application the two inputs will usually be a local oscillator and an IF oscillator, as in Figure 10.16b. The IF signal would be modulated with the desired information signal. Then the output will be \( \omega_0 \pm \omega_i \), where \( \omega_i \) is the IF frequency. The frequency \( \omega_0 + \omega_i \) is called the upper sideband (USB), while \( \omega_0 - \omega_i \) is called the lower sideband (LSB). Double sideband (DSB) modulation retains both sidebands, while single sideband (SSB) modulation removes one of the sidebands by filtering or by using an image rejection mixer (also called a single sideband modulator).

Mixer design involves impedance matching the three ports, which is complicated by the fact that several frequencies and their harmonics are involved. Undesired harmonic power can be dissipated in resistive terminations, or blocked with reactive terminations. Resistive loads increase the loss of the mixer, and reactive loads are usually very frequency sensitive. An important figure of merit for a mixer is the conversion loss, defined as

\[ L_C = 10 \log \frac{\text{available RF input power}}{\text{IF output power}} \text{ dB}. \]

10.39

Practical mixers usually have a conversion loss between 4 and 7 dB. One factor that strongly affects the conversion loss of a mixer is the local oscillator signal (or pump) power level; minimum conversion loss usually occurs for LO powers between 0 and 10 dBm. This power level is large enough to violate the small-signal approximation of (10.29), so results using such a model may not be very accurate. Precise design requires numerical solution of the nonlinear equation that describes the diode characteristics [4].

Because a mixer is often the first or second component in a receiver system, its noise characteristics can be of critical importance. When specifying the noise figure of a mixer (or a receiver that uses a mixer), a distinction must be made as to whether the input is a single sideband signal or a double sideband signal. This is because the mixer will produce an IF output for two RF frequencies \( (\omega_0 \pm \omega_i) \), and therefore collect noise power at both frequencies. When used with a DSB input, the mixer will have desired signals at both RF frequencies, while an SSB input provides the desired signal only at one of these frequencies. Thus the DSB noise figure will be 3 dB lower than the SSB noise figure.
Besides conversion loss, there are several other characteristics that describe mixer performance. Impedance matching at the RF and LO inputs is important for good signal sensitivity and noise figure. In many applications it is desirable to have good isolation between the RF and LO ports so that, for example, LO power will not be radiated out the receive antenna. Other factors include the cancellation of AM noise from the LO, and suppression of higher-order harmonics. The single-ended mixer performs reasonably well in terms of all these characteristics, but the mixer designs discussed below can be used to obtain substantially better performance for some specific characteristics.

**Balanced Mixer**

A balanced mixer combines two or more identical single-ended mixers with a 3 dB hybrid junction (90° or 180°) to give either better input SWR or better RF/LO isolation. The balanced mixer can also give cancellation of AM noise from the local oscillator. Figure 10.18 shows a photograph of a microstrip circuit that contains several balanced mixers.

The circuit for a balanced mixer is shown in Figure 10.19; it consists of two single-ended mixers with matched characteristics, driven with a 3 dB coupler. Although not shown, the single-ended mixers will require matching and bias networks. We first consider the case where a small random noise voltage, \( v_n(t) \), is superimposed on the local oscillator signal. Then the RF and LO voltages at the input of the hybrid can be expressed as

\[
v_{RF}(t) = v_r \cos \omega_r t, \quad 10.40
\]

\[
v_{LO}(t) = [v_0 + v_n(t)] \cos \omega_0 t, \quad 10.41
\]

where \( v_r \ll v_0 \), and \( v_n(t) \ll v_0 \). If we have a 90° hybrid, the voltages across the two diodes are

\[
v_1(t) = v_r \cos (\omega_r t - 90°) + (v_0 + v_n) \cos (\omega_0 t - 180°)
\]

\[= v_r \sin \omega_r t - (v_0 + v_n) \cos \omega_0 t, \quad 10.42a \]

\[
v_2(t) = v_r \cos (\omega_r t - 180°) + (v_0 + v_n) \cos (\omega_0 t - 90°)
\]

\[= -v_r \cos \omega_r t + (v_0 + v_n) \sin \omega_0 t. \quad 10.42b \]

The quadratic term of the diode V-I characteristic will give rise to the desired mixer products, so we will consider only this term and assume identical diodes so that diode currents can be represented as

\[
i_1 = k v_1^2, \quad 10.43a
\]

\[
i_2 = -k v_2^2, \quad 10.43b
\]

where the negative sign in (10.43b) accounts for the reversed polarity of the diodes. Using (10.42) in (10.43) gives the diode currents as

\[
i_1 = k[v_r^2 \sin^2 \omega_r t + (v_0 + v_n)^2 \cos^2 \omega_0 t - 2v_r(v_0 + v_n) \sin \omega_r t \cos \omega_0 t],
\]

\[
i_2 = -k[v_r^2 \cos^2 \omega_r t + (v_0 + v_n)^2 \sin^2 \omega_0 t - 2v_r(v_0 + v_n) \cos \omega_r t \sin \omega_0 t].
\]
FIGURE 10.18  Photograph of a 35 GHz microstrip monopulse radar receiver circuit. Three balanced mixers using ring hybrids are shown, along with three stepped-impedance low-pass filters, and six quadrature hybrids. Eight feedlines are aperture coupled to microstrip antennas on the reverse side. The circuit also contains a Gunn diode source for the local oscillator.

Courtesy of Militech Corporation, S. Deerfield, Mass.

FIGURE 10.19  Balanced mixer circuit.
After low-pass filtering, the remaining terms will be DC, noise, and IF frequency terms:

\[ i_1 = k \left( \frac{1}{2} v_r^2 + \frac{1}{2} (v_0 + v_n)^2 - v_r (v_0 + v_n) \sin \omega_i t \right), \quad 10.44a \]

\[ i_2 = -k \left( \frac{1}{2} v_r^2 + \frac{1}{2} (v_0 + v_n)^2 + v_r (v_0 + v_n) \sin \omega_i t \right), \quad 10.44b \]

where \( \omega_i = \omega_r - \omega_0 \) is the IF frequency. Combining these currents gives the IF output as

\[ i_{IF} = i_1 + i_2 = -2k v_r (v_0 + v_n) \sin \omega_i t \approx -2k v_r v_0 \sin \omega_i t, \quad 10.45 \]

since \( v_n << v_0 \). This result shows that the first-order terms in the noise voltage are canceled by the mixer, while the desired IF signals combine in phase. Practical mixers can give from 15 to 30 dB of AM noise rejection.

Now consider reflection of the input RF and LO signals from the diodes. If we have a balanced mixer with a 90° hybrid, the input RF signal will give rise to the following reflected waves (phasors) from the diodes:

\[ V_{\Gamma 1} = \Gamma V_1 = \frac{\Gamma V_r}{\sqrt{2}}, \quad 10.46a \]

\[ V_{\Gamma 2} = \Gamma V_2 = -j \frac{\Gamma V_r}{\sqrt{2}}, \quad 10.46b \]

where \( \Gamma \) is the reflection coefficient of each diode, and \( V_r \) is the phasor RF input voltage. These two reflections will then arrive and combine back at the RF and LO input ports with the following amplitudes:

\[ V_{\Gamma RF}^{\Gamma} = \frac{V_{\Gamma 1}}{\sqrt{2}} - j \frac{V_{\Gamma 2}}{\sqrt{2}} = \frac{1}{2} \Gamma V_r - \frac{1}{2} j \Gamma V_r = 0, \quad 10.47a \]

\[ V_{\Gamma LO}^{\Gamma} = \frac{V_{\Gamma 1}}{\sqrt{2}} + j \frac{V_{\Gamma 2}}{\sqrt{2}} = -\frac{1}{2} j \Gamma V_r + \frac{1}{2} j \Gamma V_r = j \Gamma V_r. \quad 10.47b \]

Thus the RF input is matched, but the reflected wave appears at the LO port. Similarly, when the LO port is driven, the reflected wave will appear at the RF port. So the RF and LO inputs of a mixer using a 90° hybrid will have good SWR characteristics, but the isolation between the RF and LO ports will be poor.

Alternatively, if a 180° hybrid is used with the RF applied to the sum port and the LO applied to the difference port, the RF waves reflected from the diodes will be

\[ V_{\Gamma 1} = V_{\Gamma 2} = \frac{\Gamma V_r}{\sqrt{2}}. \quad 10.48 \]

Then the reflections back at the sum and difference ports will be

\[ V^{\Sigma}_{\Gamma} = \frac{V_{\Gamma 1}}{\sqrt{2}} + \frac{V_{\Gamma 2}}{\sqrt{2}} = \Gamma V_r \quad 10.49a \]

\[ V^\Delta_{\Gamma} = \frac{V_{\Gamma 1}}{\sqrt{2}} - \frac{V_{\Gamma 2}}{\sqrt{2}} = 0. \quad 10.49b \]
The LO waves reflected from the diodes will be

\[ V_{T1} = -V_{T2} = \frac{\Gamma V_r}{\sqrt{2}}, \]

and the reflections back at the sum and difference ports will be

\[ V_{T1}' = \frac{V_{T1}}{\sqrt{2}} + \frac{V_{T2}}{\sqrt{2}} = 0, \]
\[ V_{T2}' = \frac{V_{T1}}{\sqrt{2}} - \frac{V_{T2}}{\sqrt{2}} = \Gamma V_r. \]

In both cases, the mismatch appears at the corresponding input port, while the RF and LO ports are isolated.

**Other Types of Mixers**

Like the balanced mixers described above, there are several other mixer circuits that can be used to enhance or reduce various modulation products and harmonics [4]. Some of these are briefly described below.

*Antiparallel diode mixer.* A circuit that is often used for subharmonically pumped mixers for millimeter wave applications uses a back-to-back pair of diodes, as shown in Figure 10.20. In operation, the local oscillator frequency is one-half of the usual LO frequency \((\omega_r - \omega_i)\), and the diode nonlinearity generates a second harmonic of the LO frequency to mix with \(\omega_r\) and produce the desired output frequency. Actually, most mixers can be used in this manner, but the antiparallel diode pair creates a symmetrical V-I characteristic that suppresses the fundamental mixing product of the RF and LO signals and leads to a better conversion loss. This configuration also suppresses AM noise from the local oscillator.

*Double-balanced mixer.* The single-ended mixer has an output consisting of all harmonic combinations of the RF and LO signals. The balanced mixer using a 180° hybrid suppresses all even harmonics of the LO. The double-balanced mixer, shown in Figure 10.21, can suppress even harmonics of both the LO and RF signals. This

![FIGURE 10.20](image-url)  

**FIGURE 10.20** Subharmonically pumped mixer using an antiparallel diode pair.
leads to a very low conversion loss. It uses two 180° hybrids, so it has good RF/LO isolation, but poor input SWR. It uses four diodes in a ring configuration, although a "star" arrangement can also be used.

*Image rejection mixer.* We have already noted that two distinct RF signals, \( \omega_r = \omega_0 \pm \omega_i \), can produce the same IF frequency, \( \omega_i \), when mixed with a local oscillator of frequency \( \omega_0 \). These two signals can be thought of as the upper and lower sidebands of \( \omega_0 \) modulated by \( \omega_i \), and are usually referred to as the real (desired) and image (undesired) mixer responses. The real response can be arbitrarily selected as either the USB or the LSB. The image rejection mixer of Figure 10.22 can be used to isolate these two responses into separate LSB (\( \omega_r = \omega_0 - \omega_i \)) and USB (\( \omega_r = \omega_0 + \omega_i \)) signals. When used as an up-convertor or modulator, this mixer can produce a single-sideband output signal.

The operation of the image rejection mixer can be explained as follows. Let the input RF signal consist of both upper and lower sidebands:

\[
v_r = v_{rI} \cos (\omega_0 + \omega_i)t + v_{rL} \cos (\omega_0 - \omega_i)t.
\]

**FIGURE 10.21** Double-balanced mixer circuit.

**FIGURE 10.22** Image rejection mixer circuit.
Then the input to the two mixers is

\[ v_r^A = \frac{v_U}{\sqrt{2}} \cos (\omega_0 + \omega_t)t + \frac{v_L}{\sqrt{2}} \cos (\omega_0 - \omega_t)t, \quad 10.53a \]

\[ v_r^B = \frac{v_U}{\sqrt{2}} \cos [(\omega_0 + \omega_t)t - 90^\circ] + \frac{v_L}{\sqrt{2}} \cos [(\omega_0 - \omega_t)t - 90^\circ]. \quad 10.53b \]

After mixing with an LO signal of \( \cos \omega_0 t \), the IF outputs of the mixers are

\[ v_i^A = \frac{k_{\text{U}}}{2\sqrt{2}} \cos \omega_t t + \frac{k_{\text{L}}}{2\sqrt{2}} \cos \omega_t t, \quad 10.54a \]

\[ v_i^B = \frac{k_{\text{U}}}{2\sqrt{2}} \cos(\omega_t t - 90^\circ) + \frac{k_{\text{L}}}{2\sqrt{2}} \cos(\omega_t t + 90^\circ). \quad 10.54b \]

Combining these two signals in the 90° hybrid at the IF output gives the top output signal as

\[ v_1 = \frac{k}{4} [v_U \cos \omega_t t + v_L \cos \omega_t t + v_U \cos(\omega_t t - 180^\circ) + v_L \cos \omega_t t] \]

\[ = \frac{k_{\text{U}}}{2} \cos \omega_t t, \quad 10.55a \]

which is the LSB component. The bottom IF output is

\[ v_2 = \frac{k}{4} [v_U \cos(\omega_t t - 90^\circ) + v_L \cos(\omega_t t + 90^\circ) \]

\[ + v_U \cos(\omega_t t - 90^\circ) + v_L \cos(\omega_t t - 90^\circ)] \]

\[ = \frac{k_{\text{U}}}{2} \sin \omega_t t, \quad 10.55b \]

which is the USB component. Image rejection or isolation ratios of 20 dB or more are typical. Table 10.2 summarizes some of the basic characteristics of the various mixers we have discussed.

**TABLE 10.2 Basic Characteristics of Some Mixers**

<table>
<thead>
<tr>
<th>Mixer Type</th>
<th>Number of Diodes</th>
<th>RF SWR</th>
<th>RF/LO Isolation</th>
<th>Third-Order Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single ended</td>
<td>1</td>
<td>Poor</td>
<td>Fair</td>
<td>Good</td>
</tr>
<tr>
<td>Balanced (90°)</td>
<td>2</td>
<td>Good</td>
<td>Poor</td>
<td>Good</td>
</tr>
<tr>
<td>Balanced (180°)</td>
<td>2</td>
<td>Fair</td>
<td>Excellent</td>
<td>Good</td>
</tr>
<tr>
<td>Double balanced</td>
<td>4</td>
<td>Poor</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
<tr>
<td>Image rejection</td>
<td>8</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
</tr>
</tbody>
</table>
Intermodulation Products

Frequency conversion in a detector or mixer is made possible through the use of a nonlinear device, such as a diode. We have seen, however, that this nonlinearity also gives rise to a number of undesired harmonics and mixer products. These spurious signals increase the conversion loss of a mixer, and can also lead to signal distortion. A similar effect can occur in amplifiers, since the active devices used for amplification (transistors, diodes, or tubes) are nonlinear. In general, a system using a nonlinear device has a voltage transfer function that can be written as a Taylor series:

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + \cdots$$  \hspace{1cm} \text{(10.56)}$$

For a detector or mixer, the $a_0$ term corresponds to the DC bias voltage, while the desired detected or mixed output is part of the $a_0 v_{in}$ term. For an amplifier, the linear $v_{in}$ term provides the desired response. The operation of a subharmonically pumped mixer depends on the $v_{in}^3$ term. Thus, depending on the application, one of these terms provides the desired output, while the remaining terms produce undesired spurious signals.

If the input to the system consists of a single frequency (or tone), say $v_{in} = \cos \omega_1 t$, then the output voltage given by (10.56) will consist of all harmonics, $m\omega_1$, of the input signal. These harmonics are classified by their order, which is equal to $m$. Thus, for an amplifier, the first-order harmonic (fundamental) is the desired response, and the presence of higher-order harmonics is called harmonic distortion. If an amplifier had a bandwidth of an octave or more, the second-order distortion product of a low-frequency signal could be in the passband of the amplifier. In a mixer application, single-tone distortion products are generally eliminated by filtering.

More serious problems arise when the input to the system consists of two relatively closely spaced frequencies (two-tone), say $v_{in} = \cos \omega_1 t + \cos \omega_2 t$. Then the output spectrum will consist of all harmonics of the form $m\omega_1 + n\omega_2$, where $m$ and $n$ may be positive or negative integers; the order of a given product is then defined as $|m| + |n|$. The $v_{in}^3$ term of (10.56) will produce harmonics at the frequencies $2\omega_1$, $2\omega_2$, $\omega_1 - \omega_2$, and $\omega_1 + \omega_2$, which are all second-order products. These frequencies are generally far away from the fundamentals $\omega_1$ and $\omega_2$, and so can easily be filtered. Such filtering may be impossible for a broadband amplifier, or receiver system, however. The $\omega_1 - \omega_2$ product is usually the desired result for a mixer. The $v_{in}^3$ term will lead to third-order products such as $3\omega_1$, $3\omega_2$, $2\omega_1 + \omega_2$, and $2\omega_2 + \omega_1$, which can be filtered, and to the products $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$, which generally cannot be filtered, even in a narrow-band system. Such products that arise from mixing two input signals are called intermodulation distortion; the third-order two-tone intermodulation products $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ are especially important because they may set the dynamic range or bandwidth of the system. Higher-order terms in (10.56) may also contribute such harmonics, but usually the dominant contributions come from the lowest-order terms.

A measure of the second- or third-order intermodulation distortion is given by the intercept points, which are points on the graph of output power versus input power for the nonlinear component or system under consideration. Such an intercept diagram is shown in Figure 10.23. A plot of output signal ($\omega_1 \pm \omega_2$ for mixer, or $\omega_1$ and $\omega_2$ for an amplifier) power versus input power has a slope of unity for small signal levels. As the input power increases, saturation sets in, causing clipping of the output waveform.
and signal distortion. This distortion manifests itself by diverting part of the input power to various harmonics. Equation (10.56) shows that the power in a second-order product varies as $v_i^2$, so the curve of output power for this product will have a slope of two. If the linear part of the small signal gain curve is extended, it will intercept the second-order product power curve at the second-order intercept point. This point can be specified by either the input or the output power at the intersection, and is a measure of the amount of second-order intermodulation distortion. The component would actually be operated well below this point. A similar intercept point is defined for third-order intermodulation distortion. Because this product is due to the $v_i^3$ term, its curve has a slope of three. Besides the fact that this type of distortion is more difficult to filter than other distortions, its intercept point usually occurs at a lower power level than the second-order intercept point. If two components having individual intercept output powers of $I_1$ and $I_2$, and gains $G_1$ and $G_2$, are connected in cascade, the total intercept output power, $I_T$, can be shown to be given as

$$I_T = \left( \frac{1}{G_2I_1} + \frac{1}{I_2} \right)^{-1}.$$ 

10.57

**POINT OF INTEREST:** The Spectrum Analyzer

A spectrum analyzer gives a frequency-domain representation of an input signal, displaying the average power density versus frequency. Thus, its function is dual to that of the oscilloscope, which displays a time-domain representation of an input signal. A spectrum analyzer is basically a sensitive receiver that tunes over a specified frequency band and gives a video output that is
proportional to the signal power in a narrow bandwidth. Spectrum analyzers are invaluable for measuring modulation products, harmonic and intermodulation distortion, noise and interference effects.

The diagram below shows a simplified block diagram of a spectrum analyzer. A microwave spectrum analyzer can typically cover any frequency band from several hundred megahertz to tens of gigahertz. The frequency resolution is set by the IF bandwidth, and is adjustable from about 100 Hz to 1 MHz. A sweep generator is used to repetitively scan the receiver over the desired frequency band by adjusting the local oscillator frequency, and to provide horizontal deflection of the display. An important part of the modern spectrum analyzer is the YIG-tuned bandpass filter at the input to the mixer. This filter is tuned along with the local oscillator, and acts as a preselector to reduce spurious intermodulation products. An IF amplifier with a logarithmic response is generally used to accommodate a wide dynamic range. Of course, like many modern test instruments, state-of-the-art spectrum analyzers often contain microprocessors to control the system and the measurement process. This improves performance and makes the analyzer more versatile, but can be a disadvantage in that the computer tends to remove the user from the physical reality of the measurement.

![Diagram of a spectrum analyzer]

**10.3 PIN DIODE CONTROL CIRCUITS**

Switches are used extensively in microwave systems, for directing signal or power flow between other components. Switches can also be used to construct other types of control circuits, such as phase shifters and attenuators. Mechanical switches can be made in waveguide or coaxial form, and can handle high powers, but are bulky and slow. PIN diodes, however, can be used to construct an electronic switching element easily integrated with planar circuitry and capable of high-speed operation. (Switching speeds of 10 nanoseconds or less are typical.) FETs can also be used as switching elements.

The PIN diode has V-I characteristics that make it a good RF switching element. When reverse biased, a small series junction capacitance leads to a relatively high diode impedance, while a forward bias current removes the junction capacitance and leaves the diode in a low impedance state [5]. Equivalent circuits for these two states are shown in Figure 10.24. Typical values for the parameters are: $C_j = 1 \, \text{pF}$, or less; $L_i = 0.5 \, \text{nH}$, or less; $R_e = 5 \, \Omega$, or less; $R_f = 1 \, \Omega$, or less. The equivalent circuits do not include