# Attitude Determination of a Land Vehicle using Inertial Measurement Units 

## By:

Brian Bleeker- bbleeker_02@yahoo.com
Rob MacMillan- cubsfan789@yahoo.com

Advised by:
Dr. In Soo Ahn

Date:

May 12, 2003

## Table of Contents

1. PROJECT SUMMARY ..... 3
2. PROJECT DESCRIPTION ..... 3-8
2.1. Inertial Measurement Unit ..... 3-4
2.2. Attitude Computer ..... 4-5
2.3. Resolution of Specific Forces ..... 4
2.4. Navigation Computer ..... 5-6
2.5. Completed Work ..... 6-7
2.5.1. Product Research ..... 6-7
2.5.2. Practice Matlab Code ..... 7
2.5.3. Attitude Computer Matlab Code ..... 7
2.6. Experimental Testing ..... $7-8$
3. TESTS AND RESULTS ..... 9-16
3.1. Initial IMU Testing ..... 9-10
3.2. Euler Angle Initialization ..... 10-12
3.3. Euler Angle Computation ..... 12
3.4. Filtering Methods ..... 12-14
3.5. Position Calculation ..... 14-16
4. STANDARDS/PATENTS ..... 16-17
4.1. Standards ..... 16-17
4.2. Patents ..... 17
5. EQUIPMENT LIST ..... 17
6. CONCLUSIONS ..... 17
APPENDIX A Practice Matlab Code: ..... 18-22
ECEF to NED Coordinates ..... 18-20
NED to ECEF Coordinates ..... 21-22
APPENDIX B Overall System Matlab Code: ..... 23-36
Earth Parameters ..... 23
Turn Rate of Earth ..... 23
Turn Rate of Navigation Frame ..... 24
Turn Rate of the Body in the Body Frame ..... 24
Skew Matrix ..... 25
Update Directional Cosine Matrix ..... 25
Gravity Computer ..... 25
Covariance Filter ..... 26-28
Cbn Initialization ..... 28-29
Euler Angle Computation ..... 29
Average/Velocity Slope Filter ..... 29-31
Navigation Equation - Determine Position ..... 31-32
Determine Latitude, Longitude, and Attitude ..... 32
Main Program ..... 32-36
REFERENCE ..... 37

## 1. Project Summary

To determine the attitude of a land vehicle with respect to navigation coordinates, angular rates and translational accelerations are measured using an inertial measurement unit (IMU). This attitude and position determination system can function as a stand-alone system or supplement a global positioning system in time when there is no line of sight to GPS satellites. Similar systems can be implemented to record the movement of the vehicle to prevent it from leaving a set course or from moving into an unstable or dangerous environment. These systems may help a driver by triggering increased traction control and air bag deployment as the accelerations and attitude angles change while driving. The block diagram of this system is shown in Fig. 1.1.1.


Figure 1.1.1 Overall Block Diagram of the system

## 2. Project Description

### 2.1. Inertial Measurement Unit

Movement of the land vehicle causes the output of the three accelerometers and three gyroscopes within the IMU to output the translational accelerations and angular rates with respect to the body axis. Gravity is shown as acceleration on one or more of the acceleration outputs depending upon how the IMU is attached to the land vehicle (see figure 2.1.1).


Figure 2.1.1 Inertial Measurement Unit

### 2.2. Attitude Computer

Euler angles or platform angles (roll, pitch, and yaw) are calculated by a series of equations that can be seen in Fig. 2.2.1 on the following page. The turn rate of the earth with respect to the navigation frame and the turn rate of the navigation frame with respect to the earth in the navigation frame are added together and multiplied by a directional cosine matrix to relate it to the body frame. In order to obtain the updated directional cosine matrix, the previous matrix is added to the change in directional cosine matrix multiplied by the skew matrix of the turn rate of the body with respect to the navigation frame in the body frame. From this new directional cosine matrix value, the Euler angles, or updated platform angles, can be calculated. When the system is first started, initial attitude angles must be entered in order to obtain the initial directional cosine matrix.

### 2.3. Resolution of Specific Forces

In order to resolve each of the translational acceleration terms as well as the angular rates, a series of filters must be used to eliminate noise present on the outputs of the inertial measurement unit. These filters will be implemented using Matlab. Disturbances that need to be filtered are output noise, accelerometer bias, angular rate bias, and acceleration with respect to gravity. After the IMU outputs are filtered, the acceleration outputs must be integrated to get velocity with respect to the body frame. Velocity can be integrated to get the position of the vehicle. The integration must also include the current attitude information, known as Euler angles, in order to yield the correct velocity and position.


Figure 2.2.1 Attitude Computer

### 2.4. Navigation Computer

In order to properly compute the attitude of the land vehicle, the attitude computer must be updated continuously as the vehicle moves. Also, gravity and Coriolis force must be properly factored in to give velocity and position estimates in the local navigation system. With these corrections the movement of the land vehicle can be fed back to the attitude
computer to obtain the attitude angle. This navigation computer will be implemented in Matlab. For the block diagram of the Navigation Computer see Fig. 2.4.1.


Figure 2.4.1 Navigation Computer

### 2.5. Completed Work

### 2.5.1. Product Research

A thorough search for literature and IMU manufacturers using Internet was done. Most manufacturers make military grade IMUs. This means that they are extremely expensive and, thus, beyond Bradley's limited capstone project budget. Two companies that sell inertial measurement units around $\$ 3,000$ were found.

Crossbow has two products near this price range. The IMU300CC is priced at $\$ 2,995.00$ and the IMU400CC is priced at $\$ 3,995.00$. The IMU400CC has a significantly smaller angular rate bias than the IMU300CC. The Crossbow company also offers a $10 \%$ educational discount. BEI Systron Donner also has an IMU product in the price range. The MotionPakII is priced at $\$ 2,500$, but its angular rate bias is more than twice the IMU300CC from Crossbow. BEI does not offer any educational discount and the data sheets for their inertial measurement units were very poor.

These products are significantly less expensive than military grade products because they are manufactured with silicon chips. These chips are prone to increased biases and random walk noise. For the purposes of this project the IMU300CC was purchased.

### 2.5.2. Practice Matlab Code

To get familiar with writing Matlab functions, sample data taken from a GPS receiver was analyzed. The data file was read into Matlab. This data was in earth-centered earth fixed coordinates. This means that the location of the receiver was measured with respect to the center of the earth. The goal of the exercise was to convert the ECEF coordinates to local navigation coordinates. Also, another function was written to convert the local navigation coordinates back to the ECEF coordinates. The Matlab code written for this exercise and the results from the code are attatched in Appendix A.

### 2.5.3. Attitude Computer Matlab Code

The attitude computer of the attitude determination system is written and functional. Each block of the diagram is written as individual functions. This will help in writing other programs so that each function can be called accordingly. The following list of code has been written and attached in Appendix B.
$>$ The turn rate of the navigation frame with respect to the earth in the navigation frame.
> The turn rate of the earth with respect to the inertial frame in the navigation frame.
$>$ The turn rate of the body frame with respect to the navigation frame in the body frame.
> Skew symmetric matrix for a 1 by 3 column vector.
$>$ Update the directional cosine matrix of the body frame in the navigation frame.
> Initialize the directional cosine matrix.
$>$ Create the current platform angles.
With all these functions written an overall main file was written to call the functions of the attitude computer. This function will also control the transfer of data between the resolution of specific forces, navigation computer and attitude computer.

### 2.6. Experimental Testing

To simulate the movements of a land-based vehicle, a tested platform will be constructed. This platform will have wheels to allow the user to push it in the desired direction to test the IMU accelerometers. The testing platform will also have a rotating axis that will
allow for testing the IMU gyroscopes. There will be a laptop computer on the testing platform to collect the inertial data. Also, a battery pack will be onboard to power the IMU. A layout for the testing platform is shown in Fig. 2.6.1 and 2.6.2.


Figure 2.6.1 Top View of Testing Platform


Figure 2.6.2 Side View of Testing Platform

## 3. Tests and Results

### 3.1. Initial IMU Testing

To download the data from the IMU, a software program called Gyroview was included with the IMU from Crossbow. This program collects the data from the six axis of the IMU along with the temperature and time and writes the data in ASCII format to a text file. This text file can be read directly into Matlab for data manipulation. The IMU has three modes with which data can be collected.

Voltage Mode: The analog sensors are sampled and converted to digital data with 1 mV resolution. The data is 12 -bit, unsigned. The data is just voltages, which would need to be converted to numerical angular rates and accelerations before we could analyze the data.

Scaled Sensor Mode: The analog sensors are sampled, converted to digital data, temperature compensated, and scaled to engineering units. The digital data represents the actual value of the quantities measured. The acceleration data is measured in units of gravity and therefore must be converted to force to be useful data. The angular rate data is measured in degrees per second.

Angle Mode: This mode acts the same as the scaled sensor mode, however it outputs the yaw, pitch, and rolls angles with other data.

In the project, the scaled sensor mode is used. The data is outputted to a text file in a spreadsheet format, which can be edited easily in Matlab. The angular mode was not used because it gives the angles of the yaw, pitch, and roll, which is unnecessary data. To begin the bias on all axis and the drift need to be analyzed. The data was collected when the IMU is in a stationary position. The data collected is shown below.


Figure 3.1.1 Accelerometer and Gyroscope Bias

From figure 3.1.1 it is determined that all of the axes of the IMU are extremely noisy and have significant bias.

### 3.2. Euler Angle Initialization

In order to initialize the Euler angles in an inertial measurement system, the data from the accelerometers and gyroscopes are compared to the known quantity of the rotation of the earth and local gravity calculated in the gravity computer. Because the data shown in figure 3.1.1 has such a large bias and is very noisy, it is impossible to determine the rotation of the earth and gravity from the given data. To test this method, the directional cosine matrix was created using this process while a 360 -degree rotation was performed on the z -axis. The result of this method is shown in figure 3.2.1 below.


Figure 3.2.1 360-degree z axis rotation(earth rotation and gravity method)
From the data shown in figure 3.2.1, the z axis does rotate 360 degrees however there is undesired rotation on both the x and y axis. This shows the failure of the earth rotation and gravity method to initialize the directional cosine matrix. To prove that is initialization method does not work in this situation the norm of the direction cosine matrix in every time step should be approximately 1 . The norm turned out to be much greater than 1 and is useless.

Cbn $=$

| 39.36 | 129.38 | 46.68 |
| :--- | :--- | :--- |
| 127.86 | -39.89 | -1.13 |
| -2.03 | -4.88 | -2.73 |

$\operatorname{Norm}(C b n)=141.89$
To initialize the directional cosine matrix to one that will work in this situation, the user must enter the initial roll, pitch and yaw angles. From the user inputs, a directional cosine matrix is formed using the following equations.
$C_{b}^{n}=C_{n}^{b T}=C_{1}^{T} C_{2}^{T} C_{3}^{T}$
Yaw - Rotation ? about the z axis, $C_{1}=\left[\begin{array}{ccc}\cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1\end{array}\right]$
Eq. 3.2.2

Pitch - Rotation ? about the y axis, $C_{2}=\left[\begin{array}{ccc}\cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta\end{array}\right]$
Eq. 3.2.3

Roll - Rotation F about the x axis, $C_{3}=\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi\end{array}\right]$
Eq. 3.2.4
$C_{b}^{n}=\left[\begin{array}{ccc}\cos \theta \cos \varphi & -\cos \phi \sin \varphi+\sin \phi \sin \theta \cos \varphi & \sin \phi \sin \varphi+\cos \phi \sin \theta \cos \varphi \\ \cos \theta \sin \varphi & \cos \phi \cos \varphi+\sin \phi \sin \theta \sin \varphi & -\sin \phi \cos \varphi+\cos \phi \sin \theta \sin \varphi \\ -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta\end{array}\right]$ Eq. 3.2.5

This method is less accurate because of the human error inherent in estimating the initial roll, pitch and yaw angles. However since the previous and more accurate method cannot be used with the IMU300CC the user entered method is the only option. To test this method, the directional cosine matrix was created using the user entered Euler angles, while a 360 -degree rotation was performed on the z -axis. The result of this method is shown in figure 3.2.2 below.


Figure 3.2.2 360 z-axis rotation (user entered method)
The latter of the two methods produces much better results. The user-entered method produces errors in the x and y -axis, with a 360 degree z -axis turn, between positive and
negative 0.5 degrees. There is, however, a significant error and further filtering on all axes is required. The filtering methods are discussed in Section 3.4.

### 3.3. Euler Angle Computation

The Euler angles are computed directly from the directional cosine matrix after every update. The angles are computed using a series of equations shown below.

Roll: $\phi=\arctan \left\{\frac{C_{32}}{C_{33}}\right\}$
Pitch: $\theta=\arcsin \left\{-C_{31}\right\}$
Yaw: $\varphi=\arctan \left\{\frac{C_{21}}{C_{11}}\right\}$
When the system is pitched to an angle of positive or negative 90 degrees, the system becomes unstable. An alternate method to compute the roll and yaw is needed when the system is close to this situation. Without this correction, the computed roll, pitch and yaw angles for a 360-degree pitch rotation is shown in Fig. 3.3.1.


Figure 3.3.1 Instability of system at (+/-)90 degrees pitch
The compensation method in the Matlab code is attached in Appendix B under the function $\operatorname{Euler}(\mathrm{cbn}, \mathrm{i})$.

### 3.4. Filtering Methods

To eliminate the noise and bias on the accelerometers and gyroscopes, statistical filters must be used. The data can be filtered in many ways. To eliminate the bias on the system an averaging filter is necessary. The averaging filter designed finds a mean of the data over the first 200 data points when the IMU is stationary. Then, the mean is
subtracted from the data to remove the bias. It then checks if the data point is within the minimum to maximum range of the first 200 data points. If the data is inside the minimum to maximum range the IMU is assumed to be stationary and the data is set to 0 . If the data is outside the minimum to maximum range the IMU is considered to be moving and just the mean is subtracted. The result of this filter is shown in figure 3.4.1


Figure 3.4.1 Result of the averaging filter
This method was effective in eliminating some of the bias and some of the noise present. Due to the large amounts of noise the averaging filter did not perform well enough and an additional filtering method is required.

An alternate way to eliminate noise from a system is a median filter witch eliminates the extremes over a window of data. The filter sets the entire window to the median point of the window of data. This method was also ineffective because the noise on the system was so large the median point became the noise extreme. The median filter gave poor results for all tests performed. In combination with the averaging filter the median filter also performed poorly.

The final filter attempted performed well, however, costs the system some accuracy. The final filter implemented is a variance statistical filter. The filter takes the variance over a specified range of data. If the variance is below a specified value, determined from testing, the IMU is considered stationary and the angular rate data is set to 0 and the accelerations are set to the gravity acting upon that axis. If the IMU is accelerating at a constant rate or rotating at a constant rate the variance of the data will be close to 0 and will be eliminated. The result of this filter is shown in figure 3.4.2 and 3.4.3 on the following page. The variance filter works very well for the angular rate data as shown in figure 3.4.3. However, the acceleration data exhibited too much noise to accurately trust that the variance was describing data and not strictly noise. An alternate filter must be used to filter the noisy accelerometer data.


Figure 3.4.2 Stationary IMU Roll, Pitch and Yaw Results using Variance Filter


Figure 3.4.3 360-Degree Yaw Rotation Unfiltered(left) and Variance Filtered(right)

### 3.5 Position Calculation

In order to convert acceleration data to velocity and position, two integrations factoring in the necessary corrections must be performed; integrating acceleration produces velocity and integrating velocity produces position. The corrections that must be accounted for when integrating velocity include: subtracting the force of gravity on the z -axis, subtracting the Coriolis effect (the rotation of the earth), and updating the change in attitude given by the directional cosine matrix. However, when integrating the acceleration twice, any bias seen on those axis will produce a quadratic function seen in the position. Also, any noise on the outputs of the accelerometers will distort the position of the vehicle. As discussed above, the filtering methods worked well for the rotation rate outputs from the three inertial gyroscopes, however, they were ineffective for the three outputs from the accelerometers. The filters were ineffective because the bias on the accelerometers has a random value when powered up and changes values as the system moves. The changing bias along with the random startup value disallows a direct subtraction of the bias at the input of the system. There is also significant noise on the accelerometers. This noise increases in size when the inertial measurement unit is moved
or rotated on any axis even if the movement or rotation is not on the axis being looked at. Both the changing bias and the increased noise can be seen in figure 3.5.1 below.


Figure 3.5.1 Changing Bias and increased noise on X -axis, No movement on X -axis however rotation on Z-axis

In figure 3.5 .1 the noise is very large which would indicate some movement in the X direction. There was however no movement in the X direction, only a rotation about the Z-axis(Yaw). Some of the increase in noise may be attributed to centripetal force, however, because the rotation of the body is fairly slow, the centripetal force should not contribute that much to the acceleration along the X -axis. Also, in figure 3.5.1, the changing bias can be seen. The bias in this case ranges from about 0.1 G to about 0.05 G . This seemingly random change in bias has no pattern and cannot be predicted. The bias change on all three accelerometer axis does not depend on the type of movement or the axis on which the movement occurs.
When integrated, the noise and the bias on the accelerometers produces an unwanted linear velocity as shown in figure 3.5.2 on the following page. Using the first 200 stationary data points on each axis, the slope of the linear velocity is computed and subtracted directly from the velocity of the entire data set on their respective axis. This provides some correction and improves the output of the position of the vehicle. The random bias on the system however is a very large problem. Shown in figure 3.5.3 and 3.5.4 on the following page are the results of moving the inertial measurement unit one meter north and then one meter south. Because the bias of the system changes, the movement north is not equivalent to the movement south. When there is no movement at all, the system shows a constant positive velocity. When integrated, this constant positive velocity becomes a linear change in position. Due to the excessive noise and random bias, position and velocity could not be determined using this inertial measurement unit.


Figure 3.5.2 Unwanted linear velocity on stationary test


Figure 3.5.3 and 3.5.4 Velocity and position of system with random bias

## 4. Standards/Patents

### 4.1 Standards

A standard search was done on the Internet. Places to purchase standards were found, but there were no free standards listings found. As of right now, standards have not been purchased for this project.

### 4.2 Patents

Using a patent research tool linked to the Bradley University Cullom Davis Library website the following patents have similarities to this project.
> 6,463,366 Attitude determination and alignment using electro-optical sensors and global navigation satellites
> 6,417,802 Integrated inertial/GPS navigation system
> $6,292,750$ Vehicle positioning method and system thereof
All of the above patents have to do with a system integrating a GPS receiver and an inertial measurement unit. For these systems the GPS acts to initialize and keep the IMU within error specs by resetting the system. This project does not depend on the error compensation abilities of the GPS systems. As of right now, there are no patents directly applicable to this project.

## 5. Equipment List

1 - Inertial Measurement Unit
1 - Laptop computer
1-12V battery
1 - Testing Platform

## 6. Conclusions

The overall goal of the project was to determine the attitude of a vehicle using the inputs from the inertial measurement unit. The project goal was completed successfully. As an extension of this project, determining the position of the inertial measurement unit was attempted for complete inertial navigation solutions. However, due to the excessive noise and random bias of the Crossbow IMU, position determination and, hence, a complete navigation solution is not possible with this IMU unless a separate sensor like a gyrocompass is used for a proper initialization. Although complete inertial navigation is not possible, this IMU can be applied to some useful short-term applications for detecting accelerations and velocities.

## APPENDIX A - Practice Matlab Code

The following is code for Matlab converting the position of a vehicle in earth-centered earth fixed coordinates to Latitude Longitude and altitude.
Figure A. 1 shows the position in ECEF coordinates Figure A. 2 shows the position in latitude, longitude and altitude

From the website http://www.geocode.com/eagle.html it was found that the approximate address where the data was take was 1701 E. Empire St. in Bloomington Il.

## ECEF To NED

```
function [lat, lon, alt] = ecef2ned(gtime, ecefpos)
ECEF2NED Convert ecef coordinates to NED coordinates and diplay.
[latitude, longitude, altitude] = ecef2ned(gtime, ECEF position)
    ECEF Position Array = [x, y, z]
    Example:
        array.x
        array.y
        array.z
    *** The array parameters must have *.x, *.y, and *.z.
Enter the time, and the x,y,z positions in ecef coordinates.
This function will display two figures:
            1: x,y,z position in ecef coordinates and
            2: latitude, longitude, and altitude
A negative longitude is the Western hemispere.
A negative latitude is in the Southern hemisphere.
%
% Written By: Brian Bleeker
% Rob MacMillan
b = size(gtime);
gtime = gtime(:,1) - gtime(1,1);
figure(1), subplot(311), plot(gtime, ecefpos.x), grid
title('ECEF X Position')
subplot(312), plot(gtime, ecefpos.y), grid
title('ECEF Y Position')
subplot(313), plot(gtime, ecefpos.z), grid
title('ECEF Z Position')
a =6378137.0; %semi-major axis
(equatorial) radius
b =6356752.3142; %semi-minor axis
(polar) radius
```

```
f = (a-b)/a;
e = sqrt(f*(2-f)); %eccentricity of
ellipsoid
len = length(ecefpos.x); %get length of data
for i = 1:len
    lon(i) = atan2(ecefpos.y(i), ecefpos.x(i)); %long = atan(y,x) -
direct
    lon(i) = lon(i)*180/pi; %convert to degrees
    h = 0; %initialize
    N = a;
    flag = 0;
    j = 0;
    p = sqrt(ecefpos.x(i)^2 + ecefpos.y(i)^2);
    sinlat = ecefpos.z(i)/(N*(1-\mp@subsup{e}{}{^2)+h); %First iteration}
    lat(i) = atan((ecefpos.z(i) +e^2*N*sinlat)/p);
    N = a/(sqrt(1 - (e^2)*(sinlat^2)));
    prevalt = (p/cos(lat(i)))-N;
    prevlat = lat(i)*180/pi;
    while (flag < 2) %do at least 100
iterations
        flag = 0;
        sinlat = ecefpos.z(i)/(N* (1-e^2)+h);
        lat(i) = atan((ecefpos.z(i)+e^2*N*sinlat)/p);
        N = a/(sqrt(1 - (e^2)*(sinlat^2)));
        alt(i) = (p/cos(lat(i)))-N;
        lat(i) = lat(i)*180/pi;
        if abs(prevalt-alt(i)) < .00000001
            flag = 1;
        end
        if abs(prevlat-lat(i)) < .00000001
            flag = flag + 1;
        end
        j = j+1;
        if j == 100
            flag = 2;
        end
        prevalt = alt(i);
        prevlat = lat(i);
    end
end
figure(2), subplot(311), plot(gtime, lon), grid
title('NED Longitude')
subplot(312), plot(gtime, lat), grid
title('NED Latitude')
subplot(313), plot(gtime, alt), grid
title('NED Altitude')
```



Figure A. 1 Position in ECEF Coordinates


Figure A. 2 position in latitude, longitude and altitude

## NED To ECEF

The following code converts North, East, Down coordinates to Earth Centered Earth Fixed (ECEF) coordinates.

```
function [ecef_pos] = ned2ecef2(ned)
% NED2ECEF2 Convert NED coordinates to ECEF coordinates and diplay.
%
% [ECEF position] = ned2ecef2(ned position)
    NED Position Array [n,e,d]
    Example:
            array.n
            array.e
            array.d
    *** The array parameters must have *.n, *.e, and *.d
%
% The NED positions are latitude longitude and altitude in decimal
degrees.
% A negative longitude is the Western hemispere.
% A negative latitude is in the Southern hemisphere.
%
% Written By: Brian Bleeker
% Rob MacMillan
%a = earth_shape; % call earth_shape to get earth data
a = 6378137 ; %a(1);
b = 6356752.3142; %a(2);
lat=ned.n*pi/180;
lon=ned.e*pi/180;
f = (a-b)/a;
e = sqrt(f*(2-f));
N = a / (sqrt(1-e^2*(sin(lat))^2));
%lat = ned.pos(1)/b + ned.geo_ref(1); % compute the current latitude
coslat = cos(lat);
sinlat = sin(lat);
coslon = cos(lon);
sinlon = sin(lon);
x = (N + ned.d)*coslat*coslon;
y (N + ned.d)*coslat*sinlon; % compute the current longitude
z = (N* (1 - e^2) + ned.d)*sinlat;
%r0 = r0 + ned.geo_ref(3)*coslat;
ecef_pos.x = x;% assign positions
ecef_pos.y = y;
ecef_pos.z = z;
```

The following sample data takes the NED coordinates for Dr. Ahn's house and converts it into ECEF coordinates. It then converts it back. From this we are able to find the error in the strictly mathematical model for processing the data.

Sample Data:

```
» ned
ned =
    n: 40.752169
    e: -89.672332
    d: }25
» ecef=ned2ecef2(ned)
ecef =
    x: 27672.3433890974
    y: -4838712.35630367
    z: 4141776.28953698
> [lat, lon, alt] = ecef2ned(1, ecef)
lat =
        40.752176473724
lon =
            -89.672332
alt =
        249.999985948205
```

The error for the longitude calculation is very close to 0 as it is strictly an inverse tangent operation. When converting the latitude, the error is on the order of $.00002 \%$, which is caused by the iteration process. The error at this altitude is on the order of $.00000006 \%$. As coordinates and altitude change the error will increase however we calculated that the error at 2400 meters is approximately 2 millimeters and the error 12000 meters is approximately 33 centimeters. We feel that this is reasonable error as 12000 meters is approximately the ceiling for commercial air travel ( 35000 ft .).

## APPENDIX B - Attitude Computer Matlab Code

```
Earth Parameters
function [earth]=earth_param()
% EARTH_PARAM Returns the constant parameters of the earth for use
in different functions.
%
% [earth] = earth_param()
%
% Returns:
% a - earth equitorial radius
% b - earth polor radius
% e - eccentricity of elipsoid
% f - (a-b)/a
% O - omega - earth turn rate
%
% Written By: Brian Bleeker
% Rob MacMillan
earth.a = 6378137.0;
earth.b = 6356752.3142;
earth.f = (earth.a-earth.b)/earth.a;
earth.e = sqrt(earth.f*(2-earth.f));
earth.o = 7.292115E-5;
```


## Turn Rate of the Earth With Respect to the Inertial Frame in the Navigation Frame

```
function [win] = trEARTH_IF_NED(lat)
% TREARTH_IF_LGF Find the turn rate of the earth with respect to
the inertial
% frame in the local NED frame.
%
% [win] = trEARTH_IF_NED(Latitude)
%
% Latitude is in decimal degrees.
%
% Written By: Brian Bleeker
% Rob MacMillan
earth=earth_param;
win(1) = earth.o * cos(lat*pi/180);
win(2) = 0;
win(3) = -earth.o * sin(lat*pi/180);
win = transpose(win);
```

Turn Rate of the Navigation Frame With Respect to the Earth in the Navigation Frame

```
function [wen] = trLGF_earth(ve, vn, lat, h)
% TRLGF_EARTH Find the turn rate of the local geo frame with
respect to the earth.
%
% [wen] = trLGF_earth(Velocity east, Velocity north, Latitude,
altitude)
%
% Latitude is in decimal degrees.
%
% Written By: Brian Bleeker
% Rob MacMillan
earth=earth_param;
Rn = (earth.a(1-earth.e^2))/((1-
earth.e^2*(sin(lat*pi/180))^2)^(3/2));
Re = earth.a/((1-earth.e^2* (sin(lat*pi/180) )^2)^(1/2));
wen(1) = ve/(Re+h);
wen(2) = -vn/(Rn+h);
wen(3) = (-ve*tan(lat*pi/180))/(Re+h);
wen = transpose(wen);
```

Turn Rate of the Body Frame With Respect to the Navigation Frame in the Body Frame

```
function [wnb]=trBODY_NED_BODY(win,wen,wib,cbn)
% TRBODY_NED_BODY Find the turn rate of the vehicle with respect
to the navigaton
% frame in the body frame.
%
% [wnb] = trBODY_NED_BODY(turn rate of the earth w.r.t inertial
frame in local geo frame,
% turn rate of local geo frame w.r.t the
earth,
% raw angular data from IMU, Directional
cosine matrix)
%
%
% Written By: Brian Bleeker
% Rob MacMillan
wnb=wib-cbn*(wen+win);
```


## Skew Matrix

```
function [omeganb] = skew(wnb)
% SKEW(WNB) Derive the skew matrix of wnb
% WNB = [wnb.x,wnb,y,wnd.z]
% skew(wnb)
% 0 -wnb.z wnb.y
wnb.z 0 -wnb.x
-wnb.y wnb.x 0
%
% Written By: Brian Bleeker
% Rob MacMillan
omeganb = [0,-wnb.z,wnb.y;wnb.z,0,-wnb.x;-wnb.y,wnb.x,0];
```


## Update Directional Cosine Matrix

```
function [cbn2]=update_cbn(cbn, delt, omeganb)
    UPDATE_CBN Update the directional cosine matrix to compute
    new attitude angles.
    [cbn] = update_cbn(cbn - old values, delta time, skew(wnb))
    Written By: Brian Bleeker
        Rob MacMillan
%
cbn2 = cbn + delt[cbn*omeganb];
```


## Gravity Computer

```
function [g] = gravity(lat, h, earth)
% gravity(lat, h, earth) Derive the local gravity vector.
% Caused by the mass attraction of the Earth
and the centripetal
% acceleration caused by the Earth's
rotation.
%
% g = [g.eg, g.ng, g.g]'
% eg : the meridian deflection
% ng : the deflection perpendicular to the meridian
%
% Written By: Brian Bleeker
% Rob MacMillan
%earth = earth_param;
g = 9.780318*(1+5.3024*10^-3*(sin(lat*pi/180))^2-5.9*10^-
6*(sin(2*lat*pi/180))^2);
g = g/(1+h/earth.ro)^2;
```


## Covariance Filter

```
function [covf, w2]=cov_filter3(f,w,range,g,gtime)
% Written By: Brian Bleeker
% Rob MacMillan
len=length(w.r);
flag=[0,0,0];
for i = 1 : range + 1 : (len)
    if (i + range) < len
        temp6(i:i+range)= cov(f.x(i:i+range));
        if cov(f.x(i : (i + range))) > .01
            flag(1)=flag(1)+1;
            covf.x(i:i+range) = 1;
            f2.x(i : (i + range)) =(f.x(i : (i + range)));
            if flag(1) == 1
                tmp = cov(f.x(i:i+range));
            end
            if tmp < . 5
                covf.x(i:i+range)=-1;
                f2.x(i : (i + range)) = 0;
            end
        else
            covf.x(i:i+range) = -1;
            f2.x(i : (i + range)) = 0;
        end
        temp5(i:i+range)= cov(f.y(i:i+range));
        if cov(f.y(i : (i + range))) > .01
            flag(2)=flag(2)+1;
            covf.y(i:i+range) = 1;
            f2.y(i : (i + range)) =(f.y(i : (i + range)));
            if flag(2) == 1
                tmp2 = cov(f.y(i:i+range));
            end
            if tmp2 < . 5
                covf.y(i:i+range)=-1;
                f2.y(i : (i + range)) = 0;
            end
        else
            covf.y(i:i+range) = -1;
            f2.y(i : (i + range)) = 0;
        end
        temp4(i:i+range)= cov(f.z(i:i+range));
        if cov(f.z(i : (i + range))) > .01
            flag(3)=flag(3)+1;
            covf.z(i:i+range) = 1;
            f2.z(i : (i + range)) =(f.z(i : (i + range)));
            if flag(3) == 1
                tmp3 = cov(f.z(i:i+range));
            end
            if tmp3 < .5
                covf.z(i:i+range)=-1;
                f2.z(i : (i + range)) = 0;
            end
```

```
    else
        covf.z(i:i+range) = -1;
        f2.z(i : (i + range)) = 0;
    end
    temp1(i:i+range)= cov(w.r(i:i+range));
    if cov(w.r(i : (i + range))) > 2.8
        w2.r(i : (i + range)) = (w.r(i : (i + range)));
    else
    w2.r(i : (i + range)) = 0;
    end
    temp2(i:i+range)= cov(w.p(i:i+range));
    if cov(w.p(i : (i + range))) > 7.2
        w2.p(i : (i + range)) = (w.p(i : (i + range)));
    else
    w2.p(i : (i + range)) = 0;
    end
    temp3(i:i+range)= cov(w.y(i:i+range));
    if cov(w.y(i : (i + range))) > 1
        w2.y(i : (i + range)) = (w.y(i : (i + range)));
    else
    w2.y(i : (i + range)) = 0;
    end
else
    temp6(i : len) = cov(f.x(i : len));
    if cov(f.x(i : (len))) > . 01
        covf.x(i : len) = 1;
        f2.x(i : ( len)) = (f.x(i : (len)));
        if tmp < . 5
            covf.x(i:len)=-1;
            f2.x(i : (i + len)) = 0;
        end
    else
        covf.x(i : len) = -1;
        f2.x(i : (i + len)) = 0;
    end
    temp5(i : len) = cov(f.y(i : len));
    if cov(f.y(i : (len))) > . 01
        covf.y(i : len) = 1;
        f2.y(i : ( len)) = (f.y(i : (len)));
        if tmp2 < . 5
            covf.x(i:i+range)=-1;
            f2.y(i : (i + len)) = 0;
        end
    else
        covf.y(i : len) = -1;
        f2.y(i : (i + len)) = 0;
    end
temp4(i : len) = cov(f.z(i : len));
    if cov(f.z(i : (len))) > . 01
        covf.z(i : len) = 1;
```

```
        f2.z(i : ( len)) = (f.z(i : (len)));
        if tmp3 < . 5
            covf.x(i:i+range)=-1;
            f2.z(i : (i + len)) = 0;
        end
    else
        covf.z(i : len) = -1;
        f2.z(i : (i + len)) = 0;
    end
    temp1(i : len) = cov(w.r(i : len));
    %cov(w.r(i:len))
    if cov(w.r(i : (len))) > 2.8
        w2.r(i : (len)) = (w.r(i : (len)));
    else
        w2.r(i : (len)) = 0;
    end
    temp2(i : len) = cov(w.p(i : len));
    %cov(w.p(i:len))
    if cov(w.p(i : (len))) > 7.2
        w2.p(i : (len)) = (w.p(i : (len)));
    else
        w2.p(i : (len)) = 0;
    end
    temp3(i : len) = cov(w.y(i : len));
    %cov(w.y(i:len))
    if cov(w.y(i : (len))) > 1
        w2.y(i : (len)) = (w.y(i : (len)));
    else
        w2.y(i : (len)) = 0;
        end
    end
end
```

```
Cbn Initialization
    function [cbn] = cbninit(yaw, pitch, roll)
    % CBNINIT(yaw, pitch, roll) Derive the initial directional cosine
    matrix
    %
    % (cbn)
    % {cos.p*cos.y -cos.r*sin.y + sin.r*sin.p*cos.y
    sin.r*sin.y + cos.r*sin.p*cos.y}
    % {cos.p*sin.y cos.r*cos.y + sin.r*sin.p*sin.y
    sin.r*cos.y + cos.r*sin.p*sin.y}
    % { -sin.p sin.r*cos.p
    cos.r*cos.p }
%
% Written By: Brian Bleeker
% Rob MacMillan
yaw = yaw*pi/180;
pitch = pitch*pi/180;
roll = roll*pi/180;
```

```
cbn = [cos(pitch)*cos(yaw), -cos(roll)*sin(yaw) +
sin(roll)*sin(pitch)*cos(yaw), sin(roll)*sin(yaw) +
cos(roll)*sin(pitch)*cos(yaw);
    cos(pitch)*sin(yaw), cos(roll)*cos(yaw) +
sin(roll)*sin(pitch)*sin(yaw), -sin(roll)*cos(yaw) +
cos(roll)*sin(pitch)*sin(yaw);
    -sin(pitch), sin(roll)*}\operatorname{cos(pitch), cos(roll)*cos(pitch)];
```


## Euler Angle Computation

```
function euler(cbn, i)
% euler(cbn, position) Derive the euler angles from cbn
%
% Written By: Brian Bleeker
% Rob MacMillan
global angles;
angles.p(i) = (asin(-cbn(3,1)))*180/pi;
if abs(angles.p(i)-90) <= 1
    if rem(i,2)==0
        angles.r(i) = angles.r(i-1);
        angles.y(i) = atan2((cbn (2,3)-
cbn(1,2)),(cbn(1,3)+cbn(2,2)))*180/pi + angles.r(i);
        else
            angles.y(i)=angles.y(i-1);
            angles.r(i) = -atan2((cbn(2,3)-
cbn(1,2)),(cbn(1,3)+cbn(2,2)))*180/pi + angles.y(i);
    end
elseif abs(angles.p(i)+90) <= 1
    if rem(i,2)==0
        angles.r(i) = angles.r(i-1);
        angles.y(i) = atan2((cbn (2,3)+cbn(1,2)),(cbn(1,3)-
cbn(2,2)))*180/pi - angles.r(i);
    else
        angles.y(i)= angles.y(i-1);
        angles.r(i) = atan2((cbn (2,3)+cbn(1,2)),(cbn(1,3)-
cbn(2,2)))*180/pi - angles.y(i);
    end
else
        angles.y(i) = (atan2(cbn(2,1),cbn(1,1)))*180/pi;
        angles.r(i) = (atan2(cbn(3,2),cbn(3,3)))*180/pi;
end
```


## Average/Velocity Slope Filter

```
function [ave, mx, mn, slope] = average(acc,w,g,cbn,delt)
% Written By: Brian Bleeker
% Rob MacMillan
temp_x = 0;
temp_y = 0;
```

```
temp_z = 0;
temp_wr = 0;
temp_wp = 0;
temp_wy = 0;
mx.x = -10;
mn.x = 10;
mx.y = -10;
mn.y = 10;
mx.z = -10;
mn.z = 10;
mx.wr = -10;
mn.wr = 10;
mx.wp = -10;
mn.wp = 10;
mx.wy = -10;
mn.wy = 10;
for (i = 1:400)
    a = [acc.x(i);acc.y(i);acc.z(i)];
    a = cbn * a;
        temp_x = a(1) + temp_x;
    temp_y = a(2) + temp_y;
    temp_z = a(3) + temp_z;
    temp_wr = w.r(i) + temp_wr;
    temp_wp = w.p(i) + temp_wp;
    temp_wy = w.y(i) + temp_wy;
    if a(1) < mn.x
        mn.x = a(1);
    elseif a(1) > mx.x
            mx.x = a(1);
    end
    if a(2) < mn.y
        mn.y = a(2);
    elseif a(2) > mx.y
                mx.y = a(2);
    end
    if a(3) < mn.z
        mn.z = a(3);
    elseif a(3) > mx.z
                mx.z = a(3);
    end
    if w.r(i) < mn.wr
        mn.wr = w.r(i);
    elseif w.r(i) > mx.wr
            mx.wr = w.r(i);
        end
    if w.p(i) < mn.wp
        mn.wp = w.p(i);
    elseif w.p(i) > mx.wp
            mx.wp = w.p(i);
        end
    if w.y(i) < mn.wy
        mn.wy = w.y(i);
    elseif w.y(i) > mx.wy
            mx.wy = w.y(i);
    end
    a(3)=a(3)-g;
```

```
if i > 1
vel.x(i) = a(1)*delt + vel.x(i-1);
vel.y(i) = a(2)*delt + vel.y(i-1);
vel.z(i) = a(3)*delt + vel.z(i-1);
else
vel.x(i) = a(1)*delt;
vel.y(i) = a(2)*delt;
vel.z(i) = a(3)*delt;
end
end
figure(100000);plot(vel.x)
figure(100001);plot(vel.y)
figure(100002);plot(vel.z)
ave.x = temp_x/200;
ave.y = temp_y/200;
ave.z = temp_z/200-g;
ave.wr = temp_wr/200;
ave.wp = temp_wp/200;
ave.wy = temp_wy/200;
mx.z = mx.z-g;
mn.z = mn.z-g;
slope.n = vel.x(400)/400;
slope.e = vel.y(400)/400;
slope.d = vel.z(400)/400;
```


## Navigation Equation - Determine Position

```
function [pos, vel] = position_1(f, v, lat, h, g, delt, earth, slope)
% [position, velocity] = position(force, velocity, lat, height,
gravity, change in time, earth)
%
% The Navigation Equation
%
% Written By: Brian Bleeker
% Rob MacMillan
%earth = earth_param;
%rate of change of velocity
acc.n = f.n - 2*earth.o*v.e*sin(lat*pi/180)+(v.n*v.d-
(v.e)^2*tan(lat*pi/180))/(earth.ro + h) ;%+ .001225;%- slope.n;
acc.e = f.e +
2*earth.o*(v.n*sin(lat*pi/180)+v.d*cos(lat*pi/180))+(v.e/(earth.ro+h))*
(v.d + v.n*tan(lat*pi/180));% - .00025;%- slope.e;
acc.d = f.d - 2*earth.o*v.e*cos(lat*pi/180)-
(((v.e)^2+(v.n)^2)/(earth.ro+h)) - g;% - .0005;% - slope.d;
%change rate of change of velocity into acceleration and integrate to
get velocity: v/t*t
if (abs(acc.n*delt) < .0005)
    vel.n = v.n;% - slope.n + acc.n*delt;% - .00005;
else
    vel.n = acc.n*delt + v.n - slope.n-
.0003125;%.0005;%.000125;%.000265;
```

```
end
vel.e = acc.e*delt + v.e - slope.e;
vel.d = acc.d*delt + v.d - slope.d;
%integrate velocity get position
pos.n = vel.n*delt;
pos.e = vel.e*delt;
pos.d = vel.d*delt;
```


## Latitude, Longitude, and Altitude Position Determination

```
function [pos] = position2(v, lat, h, delt, earth)
% [position] = position2(velocity, lat, height, earth)
%
% pos.latitude(i)
% .longitude(i)
% .height
%
% Written By: Brian Bleeker
% Rob MacMillan
%earth = earth_param
pos.lat = v.n/(earth.ro + h);
pos.lon = v.e*sec(lat*pi/180)/(earth.ro + h);
pos.h = v.d;
pos.lat = pos.lat*delt;
pos.lon = pos.lon*delt;
pos.h = pos.h*delt;
```


## Main Program

```
clear global
clear all
global angles;
[gtime, w.r, w.p, w.y, f.x, f.y, f.z, temp, timecounts]...
    =textread('C:\temp\IMU\lab8\data2\meter_x_1_back.txt','%f %f %f %f
%f %f %f %f %f','headerlines',8);%,'delimiter',',');
b = size(gtime);
gtime = gtime(:,1) - gtime(1,1);
f.x=f.x*9.8;
f.y=f.y*9.8;
f.z=f.z*9.8;
%%%ATTITUDE COMPUTER
%Enter the roll, pitch, yaw
yaw = 0;%12.5;%77.5
pitch = 0;%-.115;%-.115;
roll = 0;%-.535;
range = 17;
lat = 40;
```

```
lon = -89.6175;
delt = 1/207;
delt2 = range*delt;
h = 607;
i = 1;
j = 1;
time = 0;
earth = earth_param(lat); %obtain earth
data
earth.ro;
g = gravity(lat, h, earth);
[covf,w] = cov_filter3(f,w,range,g,gtime); %median filter
b
win = trEARTH_IF_NED(lat, earth); %turn rate
of the earth w.r.t inertial frame in the NED frame
wib = [w.r(1)*pi/180;w.p(1)*pi/180;w.y(1)*pi/180];
%cnb = angle_adjust(acc, g, wib, lat, earth); %inital
directional cosine matrix
cnb = cbninit(yaw,pitch, roll);
cbn = cnb'; \quadoget cnb to relate NED to
body for wnb
euler(cbn, i); %obtain the euler
angles
a = [0;0;g];
a = cbn * a;
acc.n=a(1);
acc.e=a(2);
acc.d=a(3);
[ave, mx, mn, slope] = average(f,w,g,cbn,delt);
    %FILTER
%f.x = f.x - slope.n;
%f.Y = f.Y - slope.e;
%f.z = f.z - slope.d;
%[f] = acc_filter(f, ave, mx, mn, g, cnb);
%w = ang_filter(w, ave, mx, mn);
for i = 1:b
    if i == 1
            vtemp.n = 0;
            vtemp.e = 0;
            vtemp.d = 0;
            %covftemp.x = covf.x(i);
            %covftemp.y = covf.y(i);
            %covftemp.z = covf.z(i);
        %acc = acc_filter2(acc, g, covftemp, acc, g, cnb);
```

```
    [pos, vtemp] = position_1(acc, vtemp, lat, h, g, delt, earth,
slope);
    v.n(1) = vtemp.n;
    v.e(1) = vtemp.e;
    v.d(1) = vtemp.d;
    p.n(1) = pos.n;%o;
    p.e(1) = pos.e;%0;
    p.d(1) = pos.d;%0;
    epostemp = position2(vtemp, lat, h, delt, earth);
    epos.lat(1) = epostemp.lat + lat;
    epos.lon(1) = epostemp.lon + lon;
    epos.h(1) = epostemp.h + h;
        else
        euler(cbn, i) %obtain the
euler angles
    earth = earth_param(epos.lat(i-1)); %obtain
earth data
    g = gravity(epos.lat(i-1), epos.h(i-1), earth); %Calculate
new gravity
    %covftemp.x = covf.x(i);
    %covftemp.y = covf.y(i);
    %covftemp.z = covf.z(i);
    acc.n = f.x(i);
    acc.e = f.y(i);
    acc.d = f.z(i);
    %acc = acc_filter2(acc, g, covftemp, accprev, gprev, cnb);
    if rem(i,250)==0
        cnb
        acc
    end
    a = [acc.n;acc.e;acc.d];
    a = cbn * a;
    acc.n = a(1);
    acc.e = a(2);
    acc.d = a(3);
    if rem(i,250)==0
        cbn
        acc
        i
    end
    [pos, vtemp] = position_1(acc, vtemp, lat, h, g, delt, earth,
slope);
v.n(i) = vtemp.n;
v.e(i) = vtemp.e;
v.d(i) = vtemp.d;
p.n(i) = pos.n + p.n(i-1); %positions
```

```
        p.e(i) = pos.e + p.e(i-1);
        p.d(i) = pos.d + p.d(i-1);
        epostemp = position2(vtemp, epos.lat(i-1), epos.h(i-1), delt,
earth);
    epos.lat(i) = epostemp.lat + epos.lat(i-1);
        epos.lon(i) = epostemp.lon + epos.lon(i-1);
        epos.h(i) = epostemp.h + epos.h(i-1);
            wib = [w.r(i)*pi/180; w.p(i)*pi/180; w.y(i)*pi/180];
                % IMU
angular rates
    end
    win = trEARTH_IF_NED(epos.lat(i), earth); %turn rate
of the earth w.r.t inertial frame in the NED frame
    wen = trLGF_earth(vtemp, epos.lat(i), epos.h(i), earth); %turn
rate of the NED frame w.r.t earth in the NED frame
    wnb = trBODY_NED_BODY(win,wen,wib,cnb); %turn rate
Of the body w.r.t NED in the body frame
    skewwnb = skew (wnb); %skew
wnb
    cbn = update_cbn(cbn, delt, skewwnb); %update the
directional cosine matrix
    cnb = cbn';
    accprev = acc;
    gprev = g;
    end
%Roll, Pitch, Yaw
figure(1), subplot(311), plot(angles.r), grid
title('Roll - Time in seconds')
subplot(312), plot(angles.p), grid
title('Pitch - Time in seconds')
subplot(313), plot(angles.y), grid
title('Yaw - Time in seconds')
%Position (m)
figure(22), subplot(311), plot(gtime, p.n), grid
title('Position Movement North')
subplot(312), plot(gtime, p.e), grid
title('Position Movement East')
subplot(313), plot(gtime, p.d), grid
title('Position Movement Down')
%Position (m)
figure(23), subplot(311), plot(gtime, v.n), grid
title('Velocity Movement North')
subplot(312), plot(gtime, v.e), grid
title('Velocity Movement East')
```

```
subplot(313), plot(gtime, v.d), grid
title('Velocity Movement Down')
%Position (lat, lon, h)
figure(3), subplot(311), plot(gtime, epos.lat), grid
title('Position Movement North')
subplot(312), plot(gtime, epos.lon), grid
title('Position Movement East')
subplot(313), plot(gtime, epos.h), grid
title('Position Movement Down')
```


## REFERENCE

D.H. Titterton, J.L. Weston, Strapdown Inertial Navigation Technology, Peter Peregrinus Ltd. London, England, 1997.

IMU Specifications:
IMU300CC :
http://www.xbow.com/Products/Product_pdf_files/Inertial_pdf/IMU300CC.pdf
IMU400CC :
http://www.xbow.com/Products/Product_pdf_files/Inertial_pdf/IMU400CC.pdf
IMU700CA :
http://www.xbow.com/Products/Product_pdf_files/Inertial_pdf/IMU700CA.pdf
MicroPak and MicroPak II:
http://www.systron.com/

